Price Fifteen Cents.

CAMPBELL'S
CANADIAN ARITHMETIC,
IN DECIMAL CURRENCY;

OR,

"THE FIRST BOOK OF ARITHMETIC"
SUPERSEDED.

MONTREAL & TORONTO:
JAMES CAMPBELL & SON,
1866.

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Township of Brant

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TORONTO STREET.

ERRATA.

Page 83, last line, for "divisors 16 and 25," read 16 and 15.

Page 106, Example 54, 2nd, for "·4 + 2," read ·4 ÷ 2.

Page 146, for Section V., read Section VII.

Page 129, change Exercise 64 to 66; change the next Exercise, from 65 to 67, and so on, changing the number of the other Exercises to the end, to correspond with the Nos. in the Answers.

PREFACE.

In the preparation of the Elementary Arithmetic now presented to his fellow-teachers and the public, the Author has been guided by a somewhat lengthened experience in the work of the class-room, and by hints and suggestions received during a long period of familiar intercourse with teachers.

To teach the pupil *how* to perform certain arithmetical operations *on the slate*, is certainly the very *least* and *lowest* of the many objects which an author, in this department, might properly propose to aim at, in the preparation of a text book. Yet, this, unhappily, is all that is too frequently either proposed or attempted.

To aim so to construct the text book as to secure the utmost rapidity and accuracy in performing the various operations, would be rising a step higher to meet the wishes and assist the efforts of the earnest teacher; but, even this would fall far short of comprehending all that such a teacher requires and demands, to enable him to accomplish all that he proposes to effect.

To aim, in addition to this, to familiarize the pupil with the principles which shall enable him, at once, to determine, not merely *how* to perform certain operations in obedience to orders, but to determine, in every case, *which* of the various operations *should be performed*, would be rising still higher in the effort to relieve the teacher of an unnecessary tax upon his time and ingenuity to supply the defects, or adjust the disproportions of the text book. (See p. 70, 71, &c.)

To aim, as far as possible, to lessen the peculiarly unpleasant enchantment which distance lends to the view of arithmetical rules and operations already passed over, and avert the obscurity and confusion so likely to gather around the pupil if he does not frequently retrace his steps, would indicate, still more clearly, a just appreciation of the work to be accomplished.

But this is not all, an enlightened and comprehensive survey of the whole ground to be occupied should lead an author still further to anticipate the purposes and wants of the teacher. It should certainly determine him,

(though, so far as the author is aware, it has never before done so), to attempt to discover to the pupil to what an extent the same principle is involved in a variety of apparently distinct and different operations, placed at various distances from each other, and concealed under different names; or, in other words, to show how the various operations are bound together and stand related to each other. (See pages 51, 52, 55, 56, 60, 62, 67, 69).

Such a survey of what is expected from the teacher, should determine an author to attempt yet something more. It should lead him to adopt means to render the pupil, as far as possible, *independent* of *slate* and *pencil* in making calculations; that is, it should lead him to combine *mental exercises* in nearly equal proportions with those for the slate, throughout the whole course.

Another object of chief concern to both author and teacher, should be, *so* to mingle the SCIENCE with the ART, that the pupil cannot escape the necessity of investigating, understanding, and classifying the principles on which the rules are based. (See pages 85, 86, 87, 88, &c.)

In the preparation of an Arithmetic for the use of schools, an author is bound to keep all these important ends in view, if, for no other purpose than merely to enable the teacher to turn out good practical arithmeticians. This, however, is by no means the only, or even the *chief* object to be aimed at. *A book so constructed, will derive its highest value and importance from the consideration that a pupil cannot find his way through it without obtaining a comprehensive mental training of the very highest order, especially if he be required to explain principles and operations, and construct problems of all kinds, in the hearing of the class, to the extent recommended by the Author throughout this treatise.* (See pages 46, 73, 74, 100, &c.)

Such is the Author's theory of what constitutes a complete Elementary Arithmetic adapted to the wants of both teacher and pupil.

This theory the Author has attempted to embody and apply in the following pages, with what success he must now leave his fellow-teachers and the public to judge.

TORONTO, April, 1861.

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ARITHMETIC.

SECTION I.

NOTATION AND NUMERATION, SIMPLE ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

NOTATION AND NUMERATION.

NOTATION is the expressing of numbers by certain characters.

NUMERATION is the reading of these characters.

There are two systems of Notation in use among us, viz.: the ROMAN and the ARABIC.

THE ROMAN NOTATION.

The Roman Notation is chiefly used in inscriptions, and to denote chapters and sections of books, &c. Only seven numeral letters are used, viz.: the capital letters I, V, X, L, C, D, M.

When standing alone, the letter I. denotes one, V. denotes five, X. ten, L. fifty, C. one hundred, D. five hundred, M. one thousand. All other numbers are expressed by combinations of these letters in the following manner, viz.:

1. As often as any letter is repeated, so many times its value is repeated. Thus, I. is one, II. is two; X. is ten, XX. twenty, &c.

2. When a letter of less value is placed before a letter of greater value, it takes away its own value from the greater; but when placed after it, it adds its own value to the greater. Thus V. is five. IV. is four, and VI. is six; X. is ten, IX. is nine, and XI. is eleven, &c.

3. A line or bar placed over any letter increases its value a thousand times. Thus, V. is five, $\overline{V}$ is five thousand; X. is ten, $\overline{X}$. is ten thousand.

EXERCISE 1.

Express the following numbers by letters: Eight,—eleven, —fifteen, —nineteen,—twenty-nine,—thirty-five,—ninety-nine,—one hundred and sixty,—four hundred and forty,—five hundred and sixty-nine,—one thousand one hundred and six,—two thousand and twenty-five,—six hundred and ninety-nine,—one thousand nine hundred and twenty-five,—two thousand six hundred and eighty,—four thousand nine hundred and sixty-five,—two thousand nine hundred and sixty-one,—one thousand three hundred and forty.

THE ARABIC NOTATION.

The Arabic Notation is the method of expressing numbers by certain characters, called figures. The characters used are ten, viz.:

1	2	3	4	5	6	7	8	9	0
One.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.	Naught

The first nine are called the simple units, the significant figures, and sometimes the nine digits. The last one (0) is called the cipher or zero. It is called *naught* because it has no value of itself, but is simply used to keep the significant figures in their proper places, by filling up any place or places for which there are no significant figures.

All numbers are expressed by these ten simple characters, thus: The figures are written one after the other to the right and left of a certain point called the decimal

point, thus, 124.325. The first figure to the *left* of this point is so many *units*—it is simply four units or four things. The second figure to the left is so many tens, thus 20. is two tens, or twenty; hence 24. is twenty-four. The third figure to the left of the point is so many hundreds, thus, 100. is one hundred; hence 124. is one hundred and twenty-four. The fourth is thousands,—the fifth, ten thousands,—the sixth, hundred thousands,—the seventh is units of millions,—then after tens and hundreds of millions comes billions,—then after three places for billions comes trillions, and so on.

The first figure to the *right* of the decimal point is tenths, thus, .3 is three tenths of a whole one. The second figure is hundredths, thus, .02 is two hundredths; hence, .32 is thirty-two hundredths. The third is thousandths,—the fourth is ten thousandths; hence, .3257 is three thousand two hundred and fifty-seven ten thousandths, &c.

From what has been said, we learn that the value of any figure depends upon two things: First, what the figure itself expresses, as 2, 6, 8; and second, its distance from the decimal point, (which, when not expressed, is always understood to the right of the units figure). The farther to the *left* from the point, the *greater* its value; the farther to the *right*, the *less* its value; hence, placing ciphers to the left of whole numbers, or to the right of decimals, does not alter the value, the distance of the figures from the decimal point being unchanged. Moving from the decimal point to the left, any figure in the second place is of ten times as much value as the same figure would be in the first; and in the third it is of ten times as much as in the second; in the fourth, ten times as much as in the third; and so on, *increasing* in a *ten-fold* degree to the left. In the same way, proceeding from the decimal point to the *right* hand, the figures *decrease* — the second to the right of the point being one-tenth of the first, and the third one-tenth of the second; that is, *one* of the *tenths* being *ten* of the *hundredths*, and one of the hundredths being ten of the thousandths, and so on.

EXERCISE 2.

Write down seventy-four,—one hundred and forty,—nine hundred and ninety-nine,—seven hundred and fifty,—four tenths,—twenty-seven hundredths,—fourteen thousandths,—seven hundred and sixty thousandths,—seventy-six whole numbers and twenty-seven thousandths,—six hundred and seventy-five whole numbers and eight hundred and sixty-four thousandths,—six hundred and fifteen and six hundredths.

In reading figures, we group them into periods of three figures, (one for units, one for tens, and one for hundreds), and we give each period or group of them a separate name. Thus—

WHOLE NUMBERS. Increasing to the left.									Dec. point.	DECIMALS. Decreasing to the right.								
3rd Period, Millions.			2nd Period, Thousands.			1st Period, Units.				1st Period, Thous'dths.			2nd Period, Millionths.			3rd Period, Billionths.		
Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
								4	.	6								
							6	3	.	7	2							
						6	4	2	.	5	8	3						
					2	7	0	5	.	7	6	0	8					
				6	0	8	6	4	.	8	6	5	0	4				
			5	3	7	9	0	6	.	7	0	8	6	2	9			
		1	0	6	7	0	4	5	.	2	8	6	4	3	7	0		
	5	2	0	4	6	5	8	6	.	5	9	8	6	0	7	3	4	
6	8	5	2	4	6	7	8	4	.	4	2	6	0	4	3	7	8	5

EXERCISE 3.

Let the pupil be required to read and write this exercise in as many different ways as the figures can be read and written, the teacher illustrating on the black board, especially with reference to the decimals.

SIGNS USED IN ARITHMETIC.

$+$ named plus, signifies Addition, as $4+2$ equal 6.
$-$ named minus, signifies Subtraction, as $5-2$ equal 3.
$\times$ multiplied by, signifies Multiplication, as 4×2 equal 8.
$\div$ divided by, signifies Division, as $10\div2$ equal 5.
$=$ equal to, signifies Equality, as $2+4=6$.

: is to
:: so is
: to

signifies Proportion, as $1 : 2 :: 3 : 6$.
These figures are thus read—
1 is to 2 as 3 is to 6.

$\sqrt{}$ marks the Square root, as $\sqrt{4}=2$.
$\sqrt[3]{}$ marks the Cube root, as $\sqrt[3]{8}=2$.

ADDITION TABLE.

2 and 1 are 3	7 and 2 are 9	9 and 3 are 12
3 — 1 — 4	5 — 4 — 9	8 — 4 — 12
2 — 2 — 4	6 — 3 — 9	6 — 6 — 12
2 — 3 — 5	8 — 1 — 9	7 — 5 — 12
4 — 1 — 5	8 — 2 — 10	7 — 6 — 13
4 — 2 — 6	5 — 5 — 10	8 — 5 — 13
3 — 3 — 6	6 — 4 — 10	9 — 4 — 13
5 — 1 — 6	7 — 3 — 10	7 — 7 — 14
5 — 2 — 7	9 — 1 — 10	8 — 6 — 14
4 — 3 — 7	9 — 2 — 11	9 — 5 — 14
6 — 1 — 7	6 — 5 — 11	8 — 7 — 15
6 — 2 — 8	7 — 4 — 11	9 — 6 — 15
4 — 4 — 8	8 — 3 — 11	9 — 7 — 16
5 — 3 — 8	9 — 8 — 17	8 — 8 — 16
7 — 1 — 8		

NOTE.—This brief table embraces the addition of all the simple numbers. By committing it to memory, any numbers may be added together with the utmost rapidity and accuracy; for these are the only numbers that can come together in any addition. If, for instance, 9 and 4 are 13, then every $9+4$ must give 3 for the unit figure; hence, 29 and 4 are 33, and 89 and 4 are 93, and so on. The numbers that give the same amount being arranged together, will be found an advantage.

ADDITION.

Addition is the method of finding the sum of two or more numbers.

MENTAL EXERCISES.

EXERCISE 4.

1. A boy paid 13 cents for marbles, 4 cents for apples, and had 5 cents remaining; how many had he at first? 22

2. How many are 6 dollars, 5 dollars, 4 dollars, 3 dollars, 2 dollars, and one dollar? 21

3. James had 4 marbles, John gave him 3, George gave him 4, William gave him 5, and Thomas gave him 2; how many had he then? 18

4. A drover bought sheep as follows: of one man 15, of another 7, of another 4, of another 3, and of another 6; how many did he buy in all? 35

5. A boy bought a fish-hook for 2 cents, a line for 4 cents, and a pole for 7 cents; how many cents did he give for the whole? 13

6. A grocer sold to one man 15 pounds of sugar, to another 5 pounds, to another 8 pounds, to another 2 pounds, and to another 11 pounds, at 5 cents per pound: the men in payment handed him altogether 3 dollars; what change did he give? 41.5 .05

7. A market-man received 6 dollars for butter, 7 dollars for cheese, and 9 dollars for poultry; how many dollars did he receive in all? 22

8. A man bought 5 barrels of cider for 10 dollars, and nine bushels of apples for 5 dollars; how much did he pay for the whole? 15

N. B.—The TEACHER *is, throughout the book, supposed to supply additional and varied* MENTAL EXERCISES *whenever that heading occurs.*

EXERCISES FOR THE SLATE.

EXAMPLE.—Add together 423, 134, 267.

423 134 267 — 824	Write the numbers under each other, so that units may stand under units, tens under tens, hundreds under hundreds, &c. Draw a line under them. Add the figures in the right hand column together, thus, 7, 11, 14,*—14 units make 1 ten and 4 units.

Put down the figure 4 of the number 14. and add the 1 to the next column, thus, 1, 7, 10, 12, put down the figure 2 of the 12, and add the figure 1 to next column, thus, 1, 3, 4, 8, put down the 8. The number 824 is called the *Sum or Amount.*

*In adding a column of figures, the pupil should not be allowed to repeat each separate figure, but simply to name each *amount once,* and only *once,* as he proceeds.

1	2	2	3	1	4	3	3
2	3	0	1	5	1	2	4
3	4	6	4	2	4	6	5
6	9	8	8	8	9	11	12

2	4	6	3	4	5	3	4
1	2	4	4	3	4	7	6
3	8	2	5	6	7	8	9

	$ c.		$ c.		
12	21.15	23	14.27	21	42.286
11	12.12	24	35.86	34	23.457
23	24.02	35	43.97	75	97.983
46	57.29	82	94.00	130	164.726

$ c.			$ c.		$ c.
41.27	84.173	26	37.26	42	23.25
24.15	24.429	42	25.87	56	59.50
36.28	53.836	59	74.59	85	64.75
$[illegible]	162.438	127	$137.72	183	$147.50

NOTE.—In adding or subtracting decimals, the figures must be put down so that the decimal points [illegible] stand directly under each other.

EXERCISE 5.

(1)	(2)	(3)	(4)
$412.03 c.	245	$623.29 c.	354
346.58	325	146.03	236
427.76	678	579.42	875
$ 1186.37	1248	$ 1348.74	1465

(5)	(6)	(7)	(8)
264.64	450	547.804	856
368.003	407	653.673	479
752.74	679	865.407	627
865.593	530	276.306	894

(9)	(10)	(11)	(12)
246.006	457	$ 47.12 c.	8
78.0423	608	602.35	70
604.178	92	68.27	926
40.0764	400	720.84	47
7.2135	78	79.36	5

(13)	(14)	(15)	(16)
$5120.23 c.	4268	$3687.17 c.	2407
7142.45	2426	4215.12½	798
9687.67	4276	708.17	46
4312.54	8507	9362.28	7083
8687.21	2300	96.19	579

(17)	(18)	(19)	(20)
5126.246	2427	5036.14	780
1472.583	768	784.27	5708
6826.002	9412	6070.19	1070
9687.426	893	85.19	687
2764.158	4026	7507.17	5368
4279.325	475	687.16	759

(21)	(22)	(23)	(24)
$42674.27 c.	24785	48763.473	46536
34126.19	65843	86270.102	54263
68768.15	26879	4687.643	43986
28642.14	43658	578.170	5079
65768.29	68754	49060.492	81
74387.36	56287	18709.876	641
96728.48	65423	70471.218	98076
411094.88	351624	278540.974	248653

25. How many do $7 and $4 and $8 and $24 and $62 make?

26. How many are 42 c. and 64 c. and 40 c. and 68 c. and 79 c.?

27. How many do $67 and $79 and $93 and $104 and $65 make?

28. How many do $426 and $67 and $240 and $742 make?

29. Add together $6479 and $846 and $70 and $567 and $7426.

30. Add 742 c.+64 c.+8 c.+341 c.+804 c.+60 c.+643 c.+790 c.+806 c.

31. Add $7260+$1404+$8496+$2413+$46+$4786+$3326.

32. Add $4126+$27304+$2687+$426+$876846+$746897.

33. Add $76876+$2046+$896874+$6876874+$4268+$4276.

34. Add 367068 c.+64768 c.+94687 c.+6870 c.+2489 c.+264 c.

35. What is the amount of four hundred and sixty-three,—five thousand and sixty-four,—seventy thousand and ninety-eight,—and fifty?

36. Add together seven hundred and ninety-six,—five thousand four hundred and forty,—nine hundred and

eight,—five thousand four hundred and nine,—two hundred and two thousand and fifty,—ninety-six thousand and nine,—four hundred and one.

37. How much do the following sums of money amount to, when added together: \$79.66,—\$86.4,—\$4.6,—\$20.48,—\$468.97.

38. I saw four large baskets full of apples; in one of the baskets there were four hundred and ninety-four apples, in another three hundred and sixty-eight, in another nine hundred and eighty,—and in another four hundred and four; how many apples were there in the four baskets?

39. I gave John 12 apples, James 15, Patrick 20, and I had still 25 remaining; how many apples had I at first?

MENTAL EXERCISES.

EXERCISE 6.

1. If half a pound of tea cost 18 cents, and one pound of coffee cost 9 cents more, what would be the cost of six pounds of coffee?

2. At a fruit stall I buy oranges worth 7 cents, nuts 15 cents, and order a basket of apples at \$1.16; what change shall I get from a 2 dollar bill?

3. I bought 3 bags of salt for 18 dollars, and sold 2 of them for 7 dollars more than I gave for them; the third bag sold at the same rate; how much did I sell the whole for?

4. How many times does the hammer of a clock strike in 24 hours?

5. A man being asked his age, answered that he had passed the 19 first years of his life in America, and that he had afterwards spent 7 years in Germany, 13 years in France, 8 years in Holland and 24 years in England. How old was he?

6. In an orchard, [illegible] trees bear cherries, 28 bear peaches, 8 bear plums, and 1[illegible] bear apples. How many trees are there in the orchard?

7. A gentleman gave one boy $4.76, and two others $5.24 each. What did he give in all.

8. Mary bought a comb for 10 cents, a spool of thread for 12 cents, and a paper of needles for 8 cents. She handed the clerk 37 cents; how much change ought she to receive?

9. Two men start from the same place, and travel different ways; one at the rate of three miles an hour, and the other five miles an hour. How far apart will they be at the end of the 1st hour? How far at the end of 2nd hour, of 4th hour, of 5th hour, of 8th hour, of 9th hour?

10. The hind quarters of a cow weighed one hundred and five pounds each; the fore quarters weighed ninety-four pounds each; the hide weighed sixty-three pounds; and the tallow seventy-six pounds. What was the whole weight of the cow?

SUBTRACTION.

Subtraction is the method of finding the difference between two numbers.

MENTAL EXERCISES.

EXERCISE 7.

1. James bought 18 candies, and gave John 7 of them; how many had he left?

2. A man bought some cloth for 12 dollars, and sold it for 18 dollars; what was his gain?

3. A boy has 7 chestnuts in one hand, and 4 in the other; how many more has he in one hand than in the other, and how many in both?

4. Harry is 15 years old, and Henry 9 years old; how many years older is Harry than Henry?

5. I gave 15 dollars for a cow, and 6 dollars for a sheep; how much more was given for the cow than for the sheep? What was given for both?

6. Sampson having 9 apples, gave 4 to his mother and 3 to his sister;—for his generosity his father gave him 13 more; how many had he then?

7. A man owing 48 dollars, paid 19; what had he to pay?

29

8. A man on being asked how old he was when he was married, answered that his present age was 64 years, and that he had been married 37 years; what was his age when he was married?

9. A merchant bought a piece of cloth for 115 dollars, and sold it so as to lose 23 dollars; what did he sell it for?

EXERCISES FOR THE SLATE.

EXAMPLE.—From 6237 take 4895.

```
6237
4895
————
1342
```

Place the less number under the greater, so that units may stand under units, tens under tens, &c. Draw a line under them. Begin at the units' place, that is at the 5. Take 5 from 7 and 2 remains. Put down the 2 under the 5. Go on to the next figure, which is 9. Take 9 from 3; this cannot be done. When the lower figure is in this way, greater than the figure above, *add to the upper figure as many of the same as it takes to make one of the higher*, that is here as many of the tens as it takes to make one of the hundreds, which is 10—10 and 3 are 13. Take 9 from 13 and 4 remains. Put down 4. Whenever the upper figure has thus been increased, 1 must be added to the next lower before subtracting. Thus, add 1 to 8, which makes 9. Take 9 from 2, it cannot be done; then as before, add 10 to the 2. Now take 9 from 12, and 3 remains. Put down the 3. Add 1 to 4, it will make 5. Take 5 from 6, and 1 remains Put down the 1. The result 1342 is called the *Remainder*, the *Difference*, or the *Excess*. The number from which the subtraction is made,

viz., 6237, is called the *Minuend*. The number which is subtracted, viz. 4895, is called the *Subtrahend*.

	$ c.		$ c.	$ c.
426	647.17	754	827.86	968.12
214	423.14	621	403.27	412.86
212	224.03	133	424.59	555.26

	$ c.		$ c.	
648	498.17	783	869.83	548
411	132.28	172	217.15	213

		$ c.		
423.234	742	834.14	546	643.425
279.586	489	478.27	298	169.843
143.648	253	355.87	248	473.582

EXERCISE 8.

(1)	(2)	(3)	(4)
74603	$91020.24 c.	$41021.72 c.	40000
37684	12647.58	768.31	1001

(5)	(6)	(7)	(8)
$42681.52 c.	42890	$81000.10 c.	45301
19697.43	27601	2641.07	20009

(9)	(10)	(11)
$741026831.06 c.	$6141020.13 c.	14812.0718
278904896.60	1789068.44	7419.8648

(12)	(13)	(14)
861264981	92100.2461	$1812010.41 c.
248600989	19800.7049	898901.22

15. From seven hundred and nine thousand four hundred and twenty-seven, take two hundred and fifty-one thousand eight hundred and seventy-two.

16. From two millions two hundred and two thousand and two hundredths, take nine hundred and ninety-six thousand seven tenths.

17. What is the difference between sixty-five hundred thousandths and twenty-nine hundred thousandths?

18. How much does sixty-four thousand two hundred and four exceed six thousand two hundred and forty-nine?

19. John lent James $9071, of this sum he has received back $999; how much has James yet to pay?

20. On a cherry tree there were 2046 cherries, of these 1875 were gathered; how many remained?

21. Columbus discovered America in the year 1492; how many years is it from that time to 1836?

22. In a certain school there are 436 boys, of these only 264 can write; how many are unable to write?

23. In one of the National Schools there are 427 boys, in another there are 249; how many more are there in the one than in the other?

24. John had 202 nuts in his pocket, but there being a hole in it, he lost 96 nuts; how many had he remaining?

25. On an apple tree there were 165 apples, the wind blew off two dozen and a half; how many were left?

26. A draper bought 4786 yards of cloth, and sold 3987 yards; how many yards has he unsold?

27. What sum added to sixty-five thousand seven hundred and ninety-six, will make one million four hundred and fifty-two thousand three hundred and thirteen?

28. I was born in the year 1828; how old was I in the year 1839?

MIXED QUESTIONS.

EXERCISE 9.

1. Tom had 264 marbles; he gave 64 to James, 75 to William, and 42 to John; how many had he left?

2. A merchant had 4268 yards of cloth, on Monday he sold 146 yards, on Tuesday 97, on Wednesday 246, on Thursday 198, on Friday 364, on Saturday 497; how much cloth had he remaining?

3. Three regiments went to battle, in the first there were 968 soldiers, in the second 769, and in the third 847. There were 248 men killed in the first regiment, 368 in the second, and when the regiments returned there were only 436 men in the third; how many returned from the battle?

4. A man had a journey of 298 miles to make, the first day he walked 42 miles, the second 36 miles, the third 31 miles, the fourth 27 miles; how much farther had he to go?

5. Three vessels sailed to America with emigrants, in the first vessel there were 126 men, 96 women, and 42 children; in the second vessel there were 93 men, 37 women, and 26 children; in the third vessel there were 43 men, 24 women, and 8 children. In the first vessel three persons died; in the second two were washed overboard; the third vessel was wrecked, and all on board perished; how many got safe to America?

6. A little boy went to the Zoological Gardens to see the animals; he laid his hat on the ground, which contained 264 nuts; while his attention was engaged, the monkey stole 27 of his nuts; while he was pursuing the monkey, a squirrel made off with 16 more; how many had he remaining?

7. The population of Cork is about 108,000; of Belfast 55,000; of Liverpool 166,000; of Glasgow 203,000; by how much does the population of London exceed all three cities, the population of it being 1,776,556 in the year 1831?

8. Received on Monday $247; paid away on Tuesday $196; received on Wednesday $349; paid away on Thursday $402; received on Friday $687; paid away on Saturday $398; what money had I still remaining?

MENTAL EXERCISES.

EXERCISE 10.

1. Dr. Franklin died A. D. 1790, and was 84 years old when he died; in what year was he born?

2. John has 34 marbles, and Albert 25; how many have they both; and how many more has the one than the other?

3. Having 27 dollars in my purse, I lost 8 of them, and gave away 6 more, but afterwards a man paid me a debt of $4.50; how many had I at last?

4. Stephen, at a game of marbles, won 4 and lost 6, and then had only 8 remaining; how many had he at first?

5. A farmer had 25 sheep in one field and 15 in another; he then bought enough to make his number 56; how many did he buy?

6. A man bought a yoke of oxen for 97 dollars, their services amounted to 40 dollars, and their keeping to 13 dollars; he then sold them for 80 dollars; did he gain or lose, and how much?

7. Matthew had 9 nuts, Mary gave him 10 more, and John gave him enough to make his number 39; how many did John give him?

8. A gentleman gave 853 dollars for a carriage and two horses; the carriage alone was valued at 387 dollars; what was the value of the horses? How much more were the horses worth than the carriage?

9. A man sold a cow for 20 dollars, a calf for 4 dollars, and a sheep for 3 dollars; and in payment received a waggon worth 17 dollars;—how much remains due?

MULTIPLICATION TABLE.

Twice	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 .. 4	2 .. 6	2 .. 8	2 .. 10	2 .. 12	2 .. 14
3 .. 6	3 .. 9	3 .. 12	3 .. 15	3 .. 18	3 .. 21
4 .. 8	4 .. 12	4 .. 16	4 .. 20	4 .. 24	4 .. 28
5 .. 10	5 .. 15	5 .. 20	5 .. 25	5 .. 30	5 .. 35
6 .. 12	6 .. 18	6 .. 24	6 .. 30	6 .. 36	6 .. 42
7 .. 14	7 .. 21	7 .. 28	7 .. 35	7 .. 42	7 .. 49
8 .. 16	8 .. 24	8 .. 32	8 .. 40	8 .. 48	8 .. 56
9 .. 18	9 .. 27	9 .. 36	9 .. 45	9 .. 54	9 .. 63
10 .. 20	10 .. 30	10 .. 40	10 .. 50	10 .. 60	10 .. 70
11 .. 22	11 .. 33	11 .. 44	11 .. 55	11 .. 66	11 .. 77
12 .. 24	12 .. 36	12 .. 48	12 .. 60	12 .. 72	12 .. 84

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 .. 16	2 .. 18	2 .. 20	2 .. 22	2 .. 24
3 .. 24	3 .. 27	3 .. 30	3 .. 33	3 .. 36
4 .. 32	4 .. 36	4 .. 40	4 .. 44	4 .. 48
5 .. 40	5 .. 45	5 .. 50	5 .. 55	5 .. 60
6 .. 48	6 .. 54	6 .. 60	6 .. 66	6 .. 72
7 .. 56	7 .. 63	7 .. 70	7 .. 77	7 .. 84
8 .. 64	8 .. 72	8 .. 80	8 .. 88	8 .. 96
9 .. 72	9 .. 81	9 .. 90	9 .. 99	9 .. 108
10 .. 80	10 .. 90	10 .. 100	10 .. 110	10 .. 120
11 .. 88	11 .. 99	11 .. 110	11 .. 121	11 .. 132
12 .. 96	12 .. 108	12 .. 120	12 .. 132	12 .. 144

EXTENDED MULTIPLICATION TABLE.

13 times	14 times	15 times	16 times	17 times	18 times	19 times
2 are 26	2 are 28	2 are 30	2 are 32	2 are 34	2 are 36	2 are 38
3 .. 39	3 .. 42	3 .. 45	3 .. 48	3 .. 51	3 .. 54	3 .. 57
4 .. 52	4 .. 56	4 .. 60	4 .. 64	4 .. 68	4 .. 72	4 .. 76
5 .. 65	5 .. 70	5 .. 75	5 .. 80	5 .. 85	5 .. 90	5 .. 95
6 .. 78	6 .. 84	6 .. 90	6 .. 96	6 .. 102	6 .. 108	6 .. 114
7 .. 91	7 .. 98	7 .. 105	7 .. 112	7 .. 119	7 .. 126	7 .. 133
8 .. 104	8 .. 112	8 .. 120	8 .. 128	8 .. 136	8 .. 144	8 .. 152
9 .. 117	9 .. 126	9 .. 135	9 .. 144	9 .. 153	9 .. 162	9 .. 171

SIMPLE MULTIPLICATION.

Multiplication teaches us to find what a number will amount to, when it is repeated a number of times.

MENTAL EXERCISES.

EXERCISE 11.

1. At 7 cents a-piece, what will 9 pine-apples cost?

2. What cost 15 yards of cloth at 8 dollars a yard?

3. When 2 dimes are paid for 1 duck, what will be the cost of 8 ducks? of 10 ducks? of 12 ducks?

4. James had 9 walnuts, John twice as many lacking 8, and Joseph twice as many as both James and John + 7; how many has each, and how many have they all?

5. What cost 794 barrels of flour at 9 dollars a barrel?

6. A man bought 8 pieces of cloth, each piece containing 38 yards, at 7 dollars a yard. How many yards were there, and what did he give for the whole?

CASE 1.—*When the Multiplier does not exceed* 12.

EXAMPLE.—Multiply 53 by 7.

53
7
371

Place the number by which you are to multiply under the number to be multiplied; then say 7 times 3 make 21. Put down the 1 under the 7. Then 7 times 5 make 35, and the 2 of 21 make 37. Put down the 37. The 53 is called the *Multiplicand*; the 7 is called the *Multiplier*; and the 371 is called the *Product*. The multiplicand and the multiplier taken together are called the *Factors*; thus 53 and 7 are factors.

	$ c.		$ c.	
659	427.5	642	748.15	396
2	2	2	2	2
1318	855.0	1284	1496.30	792

$ c.		$ c.		$ c.
*486.25	968	687.32	983	758.69
3	3	4	4	5
	$ c.		$ c.	
806	793.56	378	596.36	974
5	6	7	8	9
4030	4761.36	2646	4770.88	8766
$ c.		$ c.	$ c.	$ c.
742.13	866	597.27	903.40	609.02
10	11	12	6	8

EXERCISE 12.

(1)	(2)	(3)	(4)
$4276.16 c.	$67287.43 c.	$86453.26 c.	$75268.05 c.
4	2	5	3

(5)	(6)	(7)	(8)
9468	$84076.50 c.	43256	$74873.16 c.
7	8	9	10

(9)	(10)	(11)	(12)
45687	*$96844.14 c.	63875	$47389.12 c.
11	12	9	12

* The product must have as many decimals as there are in both factors. If, for instance, there are 4 in the multiplicand, and 3 in the multiplier, then the product must have 7. If there should not be as many figures in the product as are necessary to make the required number of decimals, as many ciphers must be prefixed as are necessary to make up the required number. If, for instance,

13. Multiply 875.46 by	.4	22. Multiply $983.27 c. by	2
14.	.. 7	23.	.. 7
15.	.. 9	24.	.. 4
16.	.. .6	25.	.. .8
17.	.. 3	26.	.. .6
18.	.. 5	27.	.. 5
19.	.. .10	28.	.. 9
20.	.. 11	29.	.. 12
21.	.. .12	30.	.. 11

CASE II.—*When the Multiplier is a Composite number.**

EXAMPLE.—Multiply 436 by 32.

```
  436
    4
 ----
 1744
    8
 ----
13952
```

The multiplier, viz. 32, is formed by two factors, 4 and 8; therefore instead of multiplying by 32, you may multiply by 4, and obtain the product of 1744. Multiply this product by the other factor, 8, and you obtain 13952, the product of the 435 multiplied by 32.

EXERCISE 13.

	$ c				$ c		
31.	4264.78	×	16	37.	3687.45	×	54
32.	7436.87	×	18	38.	2468.76	×	56
33.	9687.48	×	24	39.	7849.78	×	72
34.	6748.67	×	27	40.	2040.74	×	108
35.	6430.67	×	36	41.	4368.76	×	132
36.	4264.56	×	49	42.	4968.76	×	144

the number of decimals in both factors is 7, and there are only 5 figures in the product, then two ciphers must be prefixed.

EXAMPLE:—

```
.00074
   .36
------
   444
  222
------
  2664
```

Here the product has only 4, but it requires 7; it should be .0002664.

* A composite number is the product of two factors; thus, 16 is a composite number, because formed of the factors 2 and 8, or 4 and 4; 21 is formed of 3 and 7; 27 of 3 and 9; 36 of 4 and 9, or 6 and 6, or 3 and 12.

CASE III.—*When the Multiplier contains several figures.*

EXAMPLE.—Multiply 3426 by 342.

```
   3426
    342
-------
   6852
  13704
 10278
-------
1171692
```

Place the multiplier under the multiplicand, units under units, &c. Multiply by the unit figure of the multiplier, viz. 2. Then multiply by the next figure of the multiplier, viz. 4; thus, 4 times 6 make 24, but take notice that you are to place the 4 of the 24 directly under that figure of the multiplier by which you are multiplying. Proceed in the same manner with the figure 3 of the multiplier. Then add together the products obtained.

Multiply 6487 by 230.

```
    6487
     230
--------
  194610
 12974
--------
 1492010
```

Multiply $64.87 c. by .203

```
  $64.87
    .203
--------
   19461
 12974
--------
$13.16861*
```

EXERCISE 14.

	$ c.		
1. Mult. 984.76 by	64.2	9. Mult. 65839 by	958
2. ..	7.58	10. ..	627
3. ..	2.95	11. ..	369
4. ..	.496	12. ..	426
5. ..	85.7	13. ..	704
6. ..	.4368	14. ..	8743
7. ..	789.6	15. ..	6007
8. ..	3654	16. ..	9864

17. Multiply sixty-four thousand eight hundred and fifty two, by nine hundred and eighty-seven.

* Here the .16, that is the hundreds, are cents, the remaining figures, are parts of a cent.

18. Multiply four hundred and fifty-eight thousand six hundred and ninety-four, by eight thousand and seventy-six.

19. Multiply nine hundred and eighty-six thousand seven hundred and forty, by four hundred and nine.

20. There are 8766 hours in the year; how many hours are there in 20 years?

21. A grocer sells goods to the amount of $56.57 c. per week; how much does he sell during the year?

22. In a flock of 648 sheep, how many feet were there?

23. Suppose the page of a book to contain 49 lines, and each line 47 letters; how many letters does the whole page contain?

24. In 264 dozen of wine, how many bottles are there?

25. A gentleman dying gave orders in his will that his fortune should be equally divided among his five children; each received $648.75 c.; how much money did he leave?

26. Suppose that there were in the parish 896 houses, and that each house in the parish contained five persons; what would be the population of that parish?

27. A father has five children, their food and clothing cost him five cents each day; how much does the support of the children come to in the year?

28. There were in a garden eight trees, and upon each tree there were 268 apples; how many apples were there upon all the trees?

29. There were 4768 geese plucked, and 17 quills got from each goose; how many quills were got from all?

30. There were 27 desks to be made for the school, and each desk required 29 nails; how many nails were required for all the desks?

31. In a school, there were six windows in the boys' room, and four in the girls' room; in each window there were eight panes of glass; how many panes of glass were there in all?

¶ 32. I knew two boys, one of them was lazy and lay in bed till nine, the other was an active little fellow, who rose every morning at six; how many hours did the active boy gain in a year that the other lost?

33. How often does a clock strike in a year at the rate of 156 times a day?

34. How many pins may a boy point in 6 days who works 8 hours a day, and points 16,000 pins in an hour?

CASE IV.—*When there is a Fraction in the Multiplier.*

EXAMPLE.—Multiply 672 by $27\frac{3}{4}$.

$$\begin{array}{r} 672 \\ 27\frac{3}{4} \\ \hline 4704 \\ 1344 \\ 504 = \frac{3}{4} \\ \hline 18648 \end{array}$$

After multiplying by 27, we then multiply by the $\frac{3}{4}$. (that is get $\frac{1}{4}$ of 672,) and add this to the former product.

$$\begin{array}{r} 4)672 \\ \hline 168 \\ 3 \\ \hline 504 = \frac{3}{4} \end{array}$$

We multiply by $\frac{3}{4}$, or by any other fraction, by first dividing by the *denominator*, or lower figure, (the 4 in this case.) then multiply the quotient (168) by the numerator or upper figure; or, which is the same thing, we may multiply by the numerator first, and then divide the product by the denominator. To *divide* by a fraction, do just the opposite. That is multiply by the denominator, and divide the product by the numerator.

EXAMPLE: $12 \times \frac{3}{4} = 9$, but $12 \div \frac{3}{4} = 16$

$20 \times \frac{4}{5} = ?$ $\quad 20 \div \frac{4}{5} = ?$

EXERCISE 15.

1. Mult. 48643 by	$37\frac{2}{3}$	6. Mult. 48643 by	$25\frac{8}{13}$
2.	$264\frac{7}{9}$	7.	$14\frac{1}{2}$
3.	$42\frac{11}{12}$	8.	$71\frac{1}{10}$
4.	$65\frac{4}{11}$	9.	$40\frac{5}{6}$
5.	$73\frac{3}{8}$	10.	$17\frac{5}{16}$

MENTAL EXERCISES.

EXERCISE 16.

1. Mary bought 35 quarts of milk, and on her way home spilled 4 times 2 quarts, lacking 3 quarts; how many quarts had she remaining?

2. When beef is 5 cents a pound, and pork 9 cents; how much more will 9 pounds of pork cost than 9 pounds of beef?

3. Henry is 4 feet in height and John is 5; and 5 times the sum of their heights, considered as a number, is equal to their father's age + 15 years; what is the father's age?

4. If a barrel of flour will serve 12 men 8 days, how long will it serve 1 man?

5. A man failing in trade is able to pay only 68c. on a dollar; how much can he pay on a debt of $5? How much on a debt of $20?

6. If an apple cost 2 cents, an orange three times as much lacking 4 cents, and a pine-apple three times as much as the apple and orange + 5 cents; what will be the cost of all three?

DIVISION.

Division is the method, first, of finding how often one number is contained in another; and second, of dividing any quantity into a certain number of equal parts.

MENTAL EXERCISES.

EXERCISE 17.

1. How many melons may be had for 18 cents, at 3 cents a-piece?

2. How many pine-apples at 8 cents each, can be obtained for 40 cents? for 56 cents?

3. What will 13 yards of silk cost, if 5 yards cost 45 dimes?

4. A man bought 4 barrels of flour for 20 dollars, and gave 3 of them for cider, at 3 dollars a barrel; how many barrels of cider did he get?

5. In how many days can 15 men earn as much as 3 men can in 25 days?

6. If 1 man can ride 1 mile for 4 cents, how far can 2 men ride for 80 cents?

CASE I.—*When the Divisor does not exceed* 12.

SHORT DIVISION.

EXAMPLE.—Divide 252 by 6.

$$\begin{array}{r} 6)252 \\ \hline 42 \end{array}$$

Put the numbers down according to the annexed example. Find how often the figure by which you are to divide, viz: 6 is contained in the first, or first and second figures; thus, 6 into 2, no times, then 6 into 25, 4 times and 1 over. Put down the 4 under the 5. Suppose the 1 placed before the 2, which would make it 12. Say 6 into 12, twice. Put the 2 under the 2. The number 6 is called the *Divisor*; 252 the *Dividend*; and 42 the *Quotient.*

2)4628	2)6824	3)6039	4)8408
2314	3412	2013	2102
$ c.		$ c.	
2)476.58	3)76389	4)857.36	6)76590
* $238.29	25463	$214.34	12765

* When there are decimals, *point off from the right of the quotient as many for decimals as the decimals in the*

dividend exceed those in the divisor. That is, for instance, if the dividend has 3 and the divisor 1, point off 2, &c.

EXERCISE 18.

(1)	(2) $ c.	(3)	(4) $ c.
4)27645	5)687.64	6)79687	7)806.20
(5) $ c.	(6)	(7) $ c.	(8)
8)764.26	9)28676	10)642.68	11)46267

(9)	(10) $ c.	(11)
12)76426872	8)426876.42	7(9640268
(12) $ c.	(13) $ c.	(14)
9)642687.62	12)468768.76	8)46876400
(15)	(16) $ c.	(17)
6)76002041	9)43026.01	7)41260602

18. Divide 56472689 by 2
19. " " 3
20. " " 4
21. " " 5
22. " " 6

23. Divide $680.23c. by 7
24. " " 8
25. " " 9
26. " " 10
27. " " 11

Case II.—*When the Divisor is a Composite number.*

EXAMPLE.—Divide 6789 by 28.

```
   ( 4)6789
28 {  ----
   ( 7)1697 rem'ns 1
       ----
        242 rem'ns 3
```

Two factors that produce 28, are 4 and 7 : divide, then, by 4 and by 7, as in the example. The quotient found is 242, but with two remainders, viz : 3 and 1. To obtain the complete remainder,—

RULE.—*Multiply each remainder, except the first, by the product of all the divisors previous to its own, and to these several products add the first remainder*—thus, $4 \times 3 + 1 = 13$, the result, 13, is the true remainder.

EXERCISE 19.

1.	368745	÷	54	4.	204076	÷	108
2.	246876	÷	56	5.	436876	÷	132
3.	784978	÷	72	6.	496876	÷	144

CASE III.—*When the Divisor is more than 12.*

LONG DIVISION.

The only difference between *Long* and *Short Division* is, that in *Short Division* part of the operation, (that is, the *multiplying* and *substracting*) is done *mentally*, while in *Long Division* it is all put on the slate. In order to see this, let us compare them, by doing the same question both ways.

EXAMPLE.—Divide 43176 by 8.

SHORT DIVISION.

```
8)43176
 ------
  5397
```

Here we say, *mentally*, 8 into 43, 5 times (putting down the 5) then 5 times 8 is 40, 40 from 43 leaves 3. Then we suppose the 3 that is over to be placed before the next figure 1, and again say, *mentally*, 8 into 31 will go three times, 3 times 8 are 24, from 31 leaves 7. Then 8 into 77, and so on, doing the multiplying and substracting in the mind.

Same Example by LONG DINISION

```
8)43176(53.
  40
  --
   31
   24
   --
    7
```

Here we do the very same, only putting down the *product* when we multiply, and putting down the *remainder* when we substract.

We say 8 into 43 goes 5 times, putting the 5 in the quotient, then 5 times 8 is 40, putting it down—then 40 from 43 leaves 3, putting it down, then bring the 3, which is over, and the next figure (1) together, by bringing down the 1. Then repeating the same operation, 8 into 31, goes 3 times, setting the 3 in the quotient and multiply-

ing by it, setting down the product 24, then substracting and setting down the remainder 7—then bringing down, &c. Hence the Rule for Long Division would be:

RULE FOR LONG DIVISION.

Beginning at the left of the Dividend find how many times the divisor is contained in the fewest figures that will contain it, and place the quotient figure on the right of the dividend with a line between them.

Multiply the divisor by this quotient figure and write the product under that part of the dividend taken.

Substract this product from the figures above it.

Bring down the next figure of the dividend to the right of the remainder.

Divide this as before, (placing a cipher in the quotient and bringing down the next figure if at any time the dividend will not contain the divisor,) and so on till all the figures are brought down and divided.

EXAMPLE.—Divide 614326 by 427.

OPERATION.

```
427)614326(1438 300/427
    427
    ----
    1873
    1708
    ----
     1652
     1281
     ----
      3716
      3416
      ----
       300
```

Compare this operation with the rule.

EXERCISE 20.

1.	Divide 87403 by 857		7.	420.1076	÷ 438
2.	" "	521	8.	6416879	÷ 648
3.	" "	403	9.	2864976	÷ 396
4.	" "	684	10.	2876.407	÷ 4107
5.	8427.86 ÷	78	11.	64129.30	÷ 7481
6.	9768.42 ÷	946	12.	98.00147	÷ 3076

WHEN THERE ARE DECIMALS.

When there are Decimals in the *divisor* or *dividend*, or *both*, the dividend must either have the *same* number of decimals as the divisor, or it must have *less*, or it must have *more*.

1. If the *dividend* has the same number of decimals as the divisor, *no* decimals are to be pointed off.

2. If the dividend has *less* decimals than the divisor, *add as many ciphers to the dividend as are necessary to make the number of its decimals equal to those in the divisor;* it will then be like the previous case, and, of course *no decimals are to be pointed off* from the quotient.

Lastly: if the decimals in the dividend exceed those in the divisor, *point off* from the quotient *as many as the decimals in the dividend exceed those in the divisor.* That is, for instance, if the divisor has three and the dividend three from the quotient, point off NONE. If the divisor has four and the dividend two, add two to the dividend to make them equal, and point off NONE. If the divisor has two and the dividend five, point off THREE.

13.	4078948	÷ .0008	17.	78.64126	÷ 7410
14.	7198641	÷ .2864	18.	3002602	÷ .8000
15.	364.1201	÷ 1407	19.	402026.4	÷ .0069
16.	248070.8	÷ .2600	20.	9687600	÷ .4300

21. Divide six millions seven hundred and ninety-four thousandths, by four hundred and eighty thousand six hundred and nine millionths.

22. What is the ninth of \$6037.45?

23. A ship sailed in four weeks 1262 miles; how much is that per day?

24. If a vessel contains 648 gallons of water, how long will it take to discharge it all, at the rate of .18 of a gallon a minute?

25. The population of Ireland is about eight millions, and there are about 30,000 square miles of surface; how many persons to each mile?

26. The earth is about 93 millions of miles distant from the sun; how many days would a horse take in reaching the sun, supposing he went at the rate of forty-five miles per day?

27. The rays of light come from the sun to the earth in $8\frac{1}{4}$ minutes, or 495 seconds; at what rate does light move per second, the distance from the sun to the earth being 95173000 miles?

*28. The circumference of the earth is about 25000 miles; how long would a man take to walk round it at the rate of 27 miles per day?

CASE IV.—*When there is a fraction in the divisor.*

EXAMPLE.—Divide 426 by $4\frac{2}{3}$.

OPERATION.

$$
\begin{array}{r}
4\frac{2}{3})426 \\
3 \\
\hline
14)1278(91\frac{4}{14} \\
126 \\
\hline
18 \\
14 \\
\hline
4
\end{array}
$$

Here we are to divide 246 by $4\frac{2}{3}$. We first bring both the divisor and dividend to the same name as the fraction—that is (in this instance) to thirds. In the four whole numbers of the divisor there are, of course, 12 thirds, and adding the two-thirds annexed, makes it 14 thirds. Then multiplying the dividend by 3 to bring it to thirds, we have 1278 for the dividend. As the *remainder* is always of the *same name as the dividend* the remainder 4 here, will, of course, be 4 thirds.

EXERCISE 21.

1. Divide 7534 by $23\frac{3}{4}$
2. " " $46\frac{7}{8}$
3. " " $28\frac{6}{7}$
4. " " $59\frac{3}{12}$
5. Divide 7534 by $134\frac{9}{11}$
6. " " $265\frac{17}{18}$
7. " " $564\frac{16}{19}$

MENTAL EXERCISES.

EXERCISE 22.

1. If 5 oranges are worth 1 pine-apple, and 2 pine-apples are worth 1 melon; how many oranges may be bought for 4 melons?

2. When 9 bushels of rye were worth 45 dimes, 12 bushels were given for 15 yards of cloth; what did the cloth cost a yard?

3. Gave 15 pounds of sugar for 5 pounds of butter; how much did the butter cost a pound, providing 8 pounds of sugar were worth 56 cents?

4. How many oranges, at 3 cents each, must be given for 18 lemons worth 4 cents each?

5. If 4 oranges are worth 12 cents, how many oranges must be given for 6 pine-apples, worth 12 cents each?

6. How many men can in 10 days perform the same amount of work, that 8 men can in 5 days?

SECTION II.

TABLES, REDUCTION, AND COMPOUND RULES.

TABLES.

STERLING MONEY.

4 farthings	=	1 penny.
12 pence	=	1 shilling.
20 shillings	=	1 pound.
21 shillings	=	1 guinea.

£ denotes pounds, *s.* shillings, and *d.* pence.

¼ —— one farthing, or one quarter of anything.

½ —— a half-penny, or a half of anything.

¾ —— three farthings, or three quarters of anything.

OLD CANADIAN CURRENCY.

			marked.
4 farthings	=	1 penny,	*d.*
12 pence	=	1 shilling	*s.*
5 shillings	=	1 dollar	$.
4 dollars	=	1 pound	£.

NEW CANADIAN OR DECIMAL MONEY.

The denominations are dollars and cents. The coins, at present, are cents, five cent pieces, ten cent pieces, and twenty cent pieces.

1 penny of the old equals 1⅔ cents of the new.
1⅕d. of the old equals 2 cents of the new.
⅗d of the old equals 1 cent of the new.
Hence 3d of the old equals 5 cents of the new.
6d. of the old equals 10 cents of the new.
7½d. of the old equals 12½ cents of the new.
100 cents of the old equals 1 dollar of the new, marked $.

FEDERAL OR UNITED STATES MONEY.

The denominations are eagles, dollars, dimes, cents, and mills.

TABLE.

			marked
10 mills (m)	=	1 cent	*ct.*
10 cents	=	1 dime	*d.*
10 dimes	=	1 dollar	$.
10 dollars	=	1 eagle	*E.*

AVOIRDUPOIS WEIGHT.

			marked
16 drams (*dr*)	=	1 ounce	*oz.*
16 ounces	=	1 pound	*lb.*
25 pounds *	=	1 quarter	*qr.*
4 quart'rs or 100lb	=	1 hundred w'ght	*cwt.*
20 hundredweight or 2000 pounds	=	1 ton	*T.*

14 pounds make one stone, and 8 stone 1 hundredweight of 112 lbs.

This weight is used for bread, meat, grocery, for goods in general, and for all the metals except gold and silver.

TROY WEIGHT.

			marked
24 grains (*gr.*)	=	1 pennyweight	*dwt.*
20 pennyweights	=	1 ounce	*oz.*
12 ounces	=	1 pound	*lb.*

This weight is used for gold, silver, jewels, and liquors.

APOTHECARIES' WEIGHT.

			marked
20 grains	=	1 scruple	*scr.* or ℈
3 scruples	=	1 dram	*dr.* or ʒ
8 drams	=	1 ounce	*oz.* or ℥
12 ounces	=	1 pound	*lb.* or ℔

Apothecaries use this weight in mixing their medicines; but they buy and sell by avoirdupois weight.

* 28 is still used by the Custom Houses.

3*

CLOTH MEASURE.

		marked
$2\frac{1}{4}$ inches	= 1 nail	*nl.*
4 nails	= 1 quarter	*qr.*
3 quarters	= 1 Flemish ell	*Fl. ell.*
4 quarters	= 1 yard	*yd.*
5 quarters	= 1 English ell	*E. e.*
6 quarters	= 1 French ell	*F. e.*

SQUARE OR LAND MEASURE.

		marked
144 square inches	= 1 square foot	*sq. ft.*
9 square feet	= 1 square yard	*sq. yd.*
$30\frac{1}{4}$ square yards	= 1 square perch	*sq per.*
40 square perches	= 1 rood	*rd.*
4 roods	= 1 acre	*ac.*
640 acres	= 1 square mile	*sq. ml.*

The square of any number is obtained by multiplying it by itself: 12 multiplied by 12 = 144, the square of 12.

LONG MEASURE.

		marked
12 lines	= 1 inch	*in.*
12 inches	= 1 foot	*ft.*
3 feet	= 1 yard	*yd.*
$5\frac{1}{2}$ yards	= 1 perch, pole or rod	*per.*
40 perches	= 1 furlong	*fur.*
8 furlongs	= 1 mile	*ml.*
3 miles	= 1 league	*lg.*
60 geographical miles, or $69\frac{1}{2}$ British or statute miles	= 1 degree	*dg.*
360 degrees	= the earth's circumference.	

An inch is supposed to be equal to three barley-corns in length. 4 inches make one hand, used in measuring horses.

CUBIC, OR SOLID MEASURE.

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard
40 cubic feet of rough timber, or 50 cubic feet of hewn timber	= 1 ton or load
42 cubic feet	= 1 ton of shipping
16 cubic feet	= 1 cord foot
8 cord feet, or 128 cubic feet	= 1 cord of wood

A cube is a solid figure, similar to the dice, and has six equal sides. The cube of any number is obtained by multiplying it twice by itself—thus, 12×12×12=1728, the cube of 12.

MEASURE OF CAPACITY.

		marked
4 gills	= 1 pint	*pt.*
2 pints	= 1 quart	*qt.*
4 quarts	= 1 gallon	*gal.*
2 gallons	= 1 peck	*pk.*
4 pecks	= 1 bushel	*bush.*
8 bushels	= 1 quarter	*qr.*
5 quarters	= 1 load	*ld.*

By this measure both liquids and dry goods are measured. The gill, pint, quart, gallon, are chiefly used for liquids. The peck, bushel, quarter, load, &c., are used for dry articles. The gallon contains 277,274 cubic inches.

The measure formerly called heaped measure is now, by Act of Parliament, declared illegal.

Ale, wine, and beer were formerly measured by different measures. In some places a barrel of beer contains 32, in some 34, and in others 36 gallons. A hogshead of ale was computed to contain 54 gallons, a hogshead of wine 63 gallons.

2 hogsheads make 1 pipe or butt.
2 pipes or butts make 1 tun.

TIME.

		marked
60 seconds (*sec*)	= 1 minute	*min.*
60 minutes	= 1 hour	*hr.*
24 hours	= 1 day	*da.*
7 days	= 1 week	*wk.*
12 months, or 52 weeks and 1 day, or 365 days	= 1 year	*yr.*

Every fourth year contains 366 days, and is called leap year.

DAYS IN EACH MONTH.

Thirty days hath September,
April, June and November;
All the rest have thirty-one;
February twenty-eight alone,
But in Leap-Year twenty-nine.

DIVISIONS OF THE CIRCLE.

		marked
60 seconds″	= 1 minute	*min* or ′
60 minutes	= 1 degree	*deg.* or °
30 degrees	= 1 sign	*S.*
12 signs or 360°	= 1 circle of the zodiac	*C.*

QUANTITIES.

		marked
12 articles	= 1 dozen	*doz.*
20 articles	= 1 score	*sc.*
144 articles	= 1 gross	*gr.*
24 sheets paper	= 1 quire	*qr.*
20 quires	= 1 ream	*rm.*
200 pounds	= 1 barrel of pork or beef.	
196 pounds	= 1 barrel of flour.	
14 pounds	= 1 stone.	

REDUCTION.

Reduction is the changing a higher denomination to a lower, or a lower to a higher, without altering the value.

REDUCTION DESCENDING.

CASE I.—*A higher to a lower, as pounds to pence—weeks to days—quarts to pints. Reduction descending is always done by multiplication.*

EXAMPLE.—Reduce £4 9s. 6½d. to pence.

£	s.	d.
4	9	6½
20		
89		
12		
1074		
4		
4298		

Here we first multiply the £4 by 20 to bring them to shillings, *because* it takes 20*s.* to make 1 pound. We add in the 9 shillings to get all the shillings together: this will give 89s. Then we multiply the 89*s.* by 12 to bring them to pence, *because* it takes 12 pence to make one shilling; adding in the 6*d.* this gives 1074*d.* Then we multiply the pence by 4, to bring them to farthings, because it takes 4 farthings to make 1 penny, adding in the farthings; we have now 4298 farthings. Putting this into the form of a Rule it would run thus:

RULE FOR REDUCTION DESCENDING.—*Beginning with the highest denomination, multiply it by as many of the lower, (whatever lower you wish first to bring it to) as it takes to make one of the same, adding in whatever number there may be of that lower; multiply this result as before, (by as many of the lower as it takes to make one of the same, adding in the lower), and so continue till the required denomination be obtained.*

EXAMPLE.—Reduce 4 cwt. 3 qrs. 17 lbs. to ounces.

EXAMPLE.—Reduce 16 cwt. 1 qr. 19 lbs. to ounces.

Let the pupil be required to go through these, and the following exercises orally, in the following manner, before doing them. The mental discipline and the accuracy resulting from attention to this suggestion will amply repay both teacher and pupil.

First read the question distinctly as it is in the book, then proceed, all the class attending: "Here I have hundredweights, quarters, and pounds, to be reduced to ounces,—reduction descending, done by multiplication."

"RULE.—*Beginning at the highest denomination, multiply it by as many of the lower as it takes to make one of the same,* i. e., *by as many quarters as it takes to make a hundred-weight, which is 4—then the quarters by as many pounds as it takes to make a quarter, which is 25,—then the pounds by as many ounces as it takes to make a pound, which is 16.*"

EXERCISE 23.

1. Reduce £264 9s. 10d. to pence.
2. Reduce 3 qrs. 13 lbs. 12 oz. to ounces.
3. Reduce 24lbs to pennyweights.
4. Reduce 6 wks. 3 days 14 hours, to hours.
5. Reduce 76 miles 6 fur. to perches.
6. Reduce 9 sqr. mls. 1 a. 0 r. 9 yds to square inches.
7. Reduce 47 cords of wood to cubic feet.
8. Reduce 4 pipes 1 hhd. 1 brl. 19 gals. 2 qts. to quarts.
9. Reduce 71 lbs 11 oz. 3 drs. to scruples.
10. Reduce 123 acres 17 perches to square yards.
11. Reduce 9 lbs 17 dwts. to grains.
12. Reduce 569 tons 4 cwt. 3 qrs. 17 lbs. 4 oz. 7 drams, to drams.

REDUCTION ASCENDING.

II.—*A lower to a higher, as, pence to pounds—days to weeks—inches to yards, &c. Reduction ascending is always done by Division.*

EXAMPLE.—Reduce 5760 farthings to pounds.

Farthings.
4)5760
12)1440
20)120
£6

Here we first divide the 5760 farthings by 4, to bring them to pence, *because* it takes 4 farthings to make 1 penny, this gives 1440d. Then we divide the 1440*d* by 12, to bring them to shillings, because it takes 12 pence to make one shilling—this gives 120*s*. Then we divide the 120*s*. by 20, to bring them to pounds, because there are 20*s*. in a pound—this gives £6. Hence the Rule would run thus:

RULE FOR REDUCTION ASCENDING.—*Divide the given denomination by as many of the same as it takes to make one of the higher, (whatever higher you wish first to bring it to.) Divide this as before, (by as many of the same as it takes to make one of the higher) and so on, till it is reduced up to the denomination required.*

EXERCISE 24.

Let the pupil be required to go through each of the following questions, first orally, in the same manner as suggested for the questions in Reduction descending.

1. Reduce 1427 ounces to pounds.
2. Reduce 42768 farthings to pounds.
3. Reduce 2487 grains to ounces.
4. Reduce 4786 nails to yards.
5. Reduce 4796 pecks to bushels.
6. Reduce 74697 minutes to days.
7. Reduce 714986 inches to fathoms.

8. Reduce 61479867 square miles to acres, roods, &c.
9. Reduce 667789 cubic inches to cubic yards.
10. Reduce 91666 Flemish ells to French ells.
11. Reduce 17498 cubic feet to cords.
12. Reduce 2987149 mills to eagles, dollars, dimes, &c.

REDUCTION OF THE OLD CURRENCY TO THE NEW, AND THE NEW TO THE OLD.

In reducing pounds, shillings, and pence to dollars and cents, and dollars and cents to pounds, shillings, and pence, proceed in the same manner—that is, see whether the reduction is *ascending* or *descending*, then divide or multiply accordingly, as the Rule directs.

EXAMPLE.—Reduce £24 8*s*. 6*d*. to the decimal currency.

OPERATION.

£		Cts
24×400	$=$	9600
s.		
8×20	$=$	160
d. f.		
$6 = 24 \div \frac{12}{5}$	$=$	10
		\$97.70

Here the bringing the pounds and shillings to cents is Reduction descending. We first multiply the £24 by 400, because there are 400 cents in a pound. Then multiply the 8s. by 20, because there are 20 cts. in a shilling. Then bring the pence to farthings, and reduce these farthings to cents. But cents are higher than farthings, this, therefore is Reduction ascending. We therefore divide by $2\frac{2}{5}$, or, which is the same, by $\frac{12}{5}$. These three results added together give 9770 cents. Then dividing by 100, to bring them to dollars, which is done by pointing off the two right-hand figures for cents, we have \$97.70 cents. The rule, then, would read thus:

To reduce pounds, shillings, pence and farthings to dollars and cents—

RULE.—*Multiply the pounds by 400, the shillings by 20, and (changing the pence into farthings) divide the farthings*

by $\frac{10}{3}$. Add these three results together, and point off the two right-hand figures for cents, the figures to the left of the point will be dollars.

Or some may prefer this method:

OPERATION.

£		
24×4	=	96
8÷5	=	1.60
d.		
6	=	10
		$97.70

RULE.—*Multiply the pounds by 4, and divide the shillings by 5, to get the dollars. Then to get the cents, multiply all the shillings under 5 by 20, and call the pence and farthings so many cents, at sight,* 5c. *being* 3*d.*, 10c.=6*d.* 15c.=9*d.* $12\frac{1}{2}$c.=$7\frac{1}{2}$*d.*, etc.

To reduce the dollars and cents back again to pounds, shillings, &c., just reverse the operation. Divide the dollars by 4, to get the pounds, and to get the shillings, multiply what is over from the dollars by 5, and divide the cents by 20, calling the remaining cents so many pence at sight.

EXAMPLE.—Reduce $97.70c. to pounds, shillings, &c.

OPERATION.

$	£	*s.*	
97÷4=	24	5	
c.			
70÷20=	0	3	6
	£24	8	6

Here we divide the dollars by 4, to bring it to pounds; the quotient is 24 and one dollar over,—this one dollar multiplied by 5, to bring it to shillings, gives 5s. Then divide the 70c. by 20, to bring them to shillings; the quotient is 3 and 10 cents over,—this ten cents is 6d.

EXERCISE 25.

1. In 264*l.* 9*s.* 10*d.* how many dollars and cents?

2. Reduce 364*l.* 19*s.* $9\frac{1}{2}$*d.* to farthings.

3. In 247*l.* 12*s.* $8\frac{1}{2}$*d.* how many dollars and cents?

4. How many dollars and cents are there in 276 guineas?

5. In 298 crowns, how many dollars and cents?

6. Reduce 3648 sixpences to dollars and cents.

7. In 42768 farthings how many dollars and cents?

8. How many pounds are there in 67890 shillings?

9. In 426876 farthings, how many dollars and cents?

10. How many guineas are there in 36789 shillings?

11. In 68794 pence, how many crowns?

12. How many fourpences are there in 37689 shillings?

13. In 2470*l.* how many crowns?

14. How many dollars and cents in 39076 half-crowns?

15. In 29685 twopences, how many dollars and cents?

16. In 43687 crowns, how many threepences?

17. How many fivepences are there in 4796 crowns?

18. In 76971 halfpence, how many fourpences?

19. In 798302 pounds, how many five cent pieces?

20. How many crowns are there in 7968 guineas?

21. In 79201 half guineas, how many seven shillings pieces?

22. In $276.19, how many pounds, shillings and pence.

23. In 730 dollars 14 cents, how many pounds, shillings and pence?

24. How many half-sovereigns are there in 7642 guineas?

25. Reduce 3010*l.* 11*s.* 8*d.* to farthings.

26. In 7324 guineas, how many ten cent pieces?

27. In 7690 fourpences, how many ten cent pieces?

NOTE.—The pupil should be required to prove these exercises by reducing them back again to their given denomination.

COMPOUND RULES.

COMPOUND ADDITION.

The pupil will now be able to understand, and do the Compound Rules without almost any further explanation. The truth is that the Compound and Simple Rules are precisely the same. Let us compare them in order that we may see this.

SIMPLE ADDITION.

thds.	*hds.*	*tns.*	*units.*
2	4	6	5
7	8	6	4
2	3	5	6
4	1	3	2
16	8	1	7

Units.
10)17 — 1–7 { The units reduced to tens

Tens.
10)21 — 2–1 { Tens to hundreds.

Hundreds.
10)18 — 1–8 { Hundreds to thousands.

COMPOUND ADDITION.

£	*s.*	*d.*
14	3	9
2	6	8
7	15	4
12	6	3
36	12	0

Pence.
12)24 — 2–0 { The pence reduced to shillings.

Shillings.
20)32 — 1–12 { The shillings reduced to pounds.

mls.	fur.	per.
4	6	20
6	5	13
7	4	9
6	7	12
25	7	14

Perches.

40)54
1–14 — The perches reduced to furlongs.

Furlongs.

8)23
2–7 — The furlongs reduced to miles.

TIME.

yrs.	wks.	days.
24	6	3
12	16	5
41	24	4
32	13	6
110	9	4

Days.

7)18
2–4 — The days reduced to weeks.

Weeks.

52)61(1
52
9 — The weeks reduced to years.

Here the first example (see preceding page) is what is called Simple Addition, the others Compound Addition. But they are all done by the very same rule, viz:

RULE FOR ADDITION.—*Place the numbers to be added so that thingc of the same name may stand directly under each other.*

Then begin at the right hand column or lowest denomination, add it up and divide the amount by as many of the same as it takes to make one of the next higher.*

Set down the remainder and add the quotient to the next higher, and so on till all are added.

EXERCISE 26.

(1)			(2)			(3)			(4)		
£	*s.*	*d.*	*per.*	*yds.*	*ft.*	*dys.*	*hrs.*	*min.*	*cwt.*	*qrs.*	*lbs.*
12	16	4¾	16	3	2	35	16	6	4	2	12
16	4	6½	17	4	1	24	18	14	2	3	14
64	17	2¾	24	5	0	52	12	5	6	1	7
43	12	7½	33	2	2	64	13	3	3	2	24

* If we do not actually go through the work of the division in Simple Addition, it is only because setting down the right hand figure, and carrying the others is the same as dividing by 10.

(5)				(6)			(7)			(8)		
lb.	*oz.*	*dwt.*	*grs.*	*yds.*	*qrs.*	*nls.*	*ac's*	*rds.*	*phs.*	*c.yds.*	*c.ft.*	*c.in.*
5	9	8	0	27	2	3	35	3	27	19	26	567
3	2	16	16	39	2	1	48	2	39	24	18	1468
4	6	17	0	32	3	3	620	3	15	36	11	246
1	8	19	22	47	3	2	17	1	20	39	20	1294

9. A brewer bought five bags of hops; No. 1 weighed 1 cwt. 2 qrs. 14lb.; No. 2 weighed 1 cwt. 2 qrs. 22 lb.; No. 3 weighed 1 cwt. 1 qr. 23 lb.; No. 4 weighed 1 cwt. 3 qrs. 21 lb.; No. 5 weighed 2 cwt. 2 qrs. 20 lb.; what was the weight of the whole?

10. A man rode 35 miles, 2 furlongs, 34 perches; walked 24 miles, 6 furlongs, 25 perches, 2 yards; then rode again 42 miles, 7 furlongs, 4 yards; then walked again 15 miles, 4 furlongs, 38 perches, 3 yards; what was the length of his journey?

11. Sold to one man 27 qrs. 6 bushels, 3 pecks; to another 38 qrs. 4 bushels, 2 pecks; to another 49 qrs. 6 bushels; and to another 58 qrs. 7 bushels, 3 pecks; how much did I sell in all?

12. I bought four fields; in the first there were 6 acres, 3 roods, 12 perches; in the second 7 acres 2 roods; in the third 9 acres and 13 perches; in the fourth 5 acres, 2 roods, 36 perches. How much in all?

13. The bricklayers were engaged about a house 23 weeks, 4 days, and 8 hours; the carpenters 14 weeks, 6 days, and 9 hours; the painters 12 weeks, 5 days, 7 hours, and 34 minutes; the upholsterer 5 weeks, 10 hours and 42 minutes; how long were these different workmen engaged about the house?

14. A silversmith made three dozen spoons weighing 5 lb. 9 oz. 8 dwt.; a tea-pot, weighing 3lb. 2 oz. 16 dwt. 16 grs.; two pair silver candlesticks, weighing 4 lb. 6 oz. 17 dwt.; a dozen silver forks, weighing 1 lb. 8 oz. 19 dwt. 22 grs.; what was the weight of all the articles?

15. A tailor bought four pieces of cloth; in the first

there were 27 yds. 2 qrs. 3 nls.; in the second, 39 yds. 2 qrs. 1 nl.; in the third, 32 yds. 3 qrs. 3 nls.; in the fourth, 47 yds. 3 qrs. 2 nls; how much in all?

16. A man bought a coach for £35 12s., a horse for £27 8s. 10d., and harness for £31 50s.; what did the whole cost?

17. A boat took in freight as follows: at one place 9576 lbs of butter, at another 11 tons of pork, at a third, 18 cwt. 27lbs. of coal; what was the entire freight in "short tons?"

18. A merchant bought 3 casks of oil, one held 2 hhd. 30 gals. 2 qts.; another 3 hhds, 10 gall., another 1 hhd. 13 g. 1 qt.; how much did they all hold?

19. Find the sum of 45 m. $2\frac{1}{2}$ fur. 17 p. 5 yds. 2 ft. 9 in.; and 43 m. $5\frac{1}{2}$ fur. 4 yds. 1 ft. 8 in.; and 89 m. 16 p. 3 yd. 2 ft. 5 in.

20. In one pile of wood are 37 cords 119 c. ft. 76 c. in., in another, 9 cords 104 c. ft., in a third, 48 cords 7 c. ft. 127 c. in., in a fourth, 61 cords 139 c. in.; how much wood in the 4 piles?

MENTAL EXERCISE.

EXERCISE 27.

1. A man bought one load of hay for £7 3s., and another for £6 8s. 4d.; how much did he give for both?

2. A man bought 3 bu. 3 pks. of wheat at one time; 48 bu. 3 pks. at another time; 9 bu. 1 pk. 5 qts. at a third; and 16 bu. 0 pk. 7 qts. at a fourth. How many bushels did he buy in the whole?

3. A man bought 4 bales of cotton. The first contained 4 cwt. 2 qrs. 16 lb.; the second 3 cwt. 1 qr. 14 lbs.; the third 5 cwt. 0 qr. 23 lbs.; and the fourth 4 cwt. 3 qrs. What was the weight of the whole?

4. A merchant bought 4 pieces of cloth. The first contained 18 yds. 3 qrs.; the second 23 yds. 1 qr. 3 nls; the third 25 yds.; and the fourth 16 yds. 2 qrs. 2 nls. How many yards in the whole?

5. A man bought a cask of raisins for £7 18s. 4d.; 1 lb. of coffee for 1s. 6d.; 1 cwt. of cocoa for £3 17s; 1 keg of molasses for 13s. 7d.; 1 box lemons for £1 3s.; 1 bushel of corn for 4s. 3d. How much will the whole amount to?

6. A merchant bought four pieces of cloth, each piece containing 57 yards. For the first piece he gave 235 dollars; for the second, 384 dollars; for the third, 327 dollars; and for the fourth, 486 dollars. How many yards of cloth did he buy? How much did he give for the whole?

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1. Compare *Compound* and *Simple Addition*, and show in what respect precisely they agree, and in what respect they differ.

2. Repeat the rule *for Reduction Ascending* and the rule for *Reduction Descending*.

3. Repeat the rule for *Compound Addition*, and give the reason for each part of it.

4. Point out *distinctly* the relation that exists between Addition and Multiplication.

5. Name the denominations in each of the tables, from the *lowest* to the *highest* and from the *highest* to the *lowest*, and say how many of the *lower* of each denomination it takes to make one of the *higher*.

6. Try by the watch, how quickly you can read up and down any column of *fifty figures*, simply touching each figure, *naming only the amount* as you proceed. Continue this exercise till you can read any column of 50 figures up and down, (getting the same amount both ways) in *one minute*.

COMPOUND SUBTRACTION.

Here again, as in *Addition*, we shall see by comparing the operation in each, that *Simple* and *Compound Subtraction* are both done in precisely the same way—*i. e.*, by the very same rule—so that if a person can do *Simple Subtraction*, and understands what he is doing, he can do *Compound Subtraction.*

Let us do a question in each, and compare each operation, that we may see this.

EXAMPLE 1.

SIMPLE.

	thds.	*hds.*	*tns.*	*units.*
From	7	8	1	6
Take	4	9	9	3
	2	8	2	3

EXAMPLE 2.

COMPOUND.

	£.	*s.*	*d.*
From	14	13	7
Take	12	17	4
	1	16	3

EXAMPLE 3.

SIMPLE.

	thds.	*hds.*	*tns.*	*units.*
From	9	8	7	6
Take	6	7	8	9

EXAMPLE 4.

COMPOUND.

	cwt.	*qrs.*	*lbs.*
From	16	2	12
Take	12	3	24

Here in the first example, *i. e.*, in the *Simple Subtraction*, as the lower figure 3 is less than the upper figure 6, we say 3 units from 6 units and 3 units remain, setting down the 3; and in the second example, 4 pence from seven pence and three pence remain, setting down the 3. So far they are exactly alike.

Then proceeding with the first example, we say 9 tens from 1 ten, I can't. Here, the lower figure is *greater* than the figure above. Now, in such case, we "*add to the upper figure as many of the same as it takes to make one of the higher*," (and this we do *alike*, both in *Simple* and *Compound*)—*i. e.*, in our first example as many tens as it takes to make a hundred, which is 10, and in the 2d example as many shillings as it takes to make a pound, which is 20. Proceeding with the first we say, 10 and 1 is 11; 9 from 11 and 2 remains, setting down the 2; then we add 1

to the next figure in the subtrahend before subtracting: 1 to 9 is 10; 10 from 8 I can't; then do as before, and so on. Proceeding now with the second example we say, 17 shillings from 13 I can't, then "*add to the upper figure as many of the same as it takes to make one of the higher*," just as in the *Simple—i. e.*, as many shillings as it takes to make a pound, which is 20; 20 and 13 is 33; 17 from 33 and 16 remains, &c. Surely we need not proceed further in order to see that the *Simple* and *Compound* are done in precisely the same way. Let the pupil himself do and compare the 3rd and 4th examples. A plain statement of what we have here been doing, such as the following, would be the *rule* for *Subtraction*:—

RULE FOR SUBTRACTION.—*Write down the numbers to be subtracted, the less under the greater, so that things of the same name may stand directly under each other.*

Begin at the right-hand or lowest denomination, take the lower figure from the figure above and write the remainder directly below.

If the lower figure is greater than the figure above, add to the upper figure as many of the same as it takes to make one of the next higher, then subtract the lower figure and set down the remainder. But when the minuend has been thus increased by adding to it, the next figure in the subtrahend must be increased equally by adding one to it before subtracting.

EXERCISE 28.

(1)			(2)			(3)		
£.	s.	d.	cwt.	qrs.	lbs.	lbs.	oz.	dwt.
73	10	$5\frac{1}{4}$	17	1	10	18	6	14
48	18	$9\frac{3}{4}$	10	2	27	14	9	15

(4)			(5)			(6)		
fur.	per.	yds.	yds.	qrs.	nls.	ac.	rd.	per.
7	10	1	37	3	2	42	1	10
2	19	4	18	3	2	16	2	25

7. St. Paul's bell in London weighs 5 tons 2 cwt. 1 qr. 22 lbs.; by how much does the great bell of Moscow exceed it, which weighs 198 tons 2 cwt. 1 qr.?

8. Three dozen silver table spoons weighed 5 lb. 9 oz. 8 dwt., while three dozen silver tea-spoons weighed only 1 lb. 9 oz. 16 dwt. 18 grs.; what was the difference in weight?

9. From 160 deg. 18 statute m. 210 r. 3 yds. 1 ft., take 63 deg. 25 m. 305 r. 4 yds. 2 ft.

10. A tailor, from a piece of cloth containing 37 yds. 3 qrs. 2 nls., cut off 18 yds. 3 qrs. 2 nls.; how much remained?

11. A farmer has two meadows, one containing 9 a. 3 r. 37 p., the other contains 10 a. 2 r. 25 p.; also three pastures, the first containing 12 a. 1 r. 1 p., the second containing 13 a. 3 r., and the third 6 a. 1 r. 39 p.; by how many acres does the pasture exceed the meadow land?

12. A farmer had 576 bu. 1 pk. 2 qt. of wheat; he sold 139 bu. 2 pk. 3 qt. 1 pt.; how much remained unsold?

13. Two vessels sailed for England; one of them was 9 weeks, 6 days, and 14 hours on the voyage; the other got to England in 7 weeks, 5 days, and 19 hours; how much less time did the one go in than the other?

14. Sold a merchant one quarter of beef for £2 7s. 9d.; one cheese for 9s. 7d.; for which I received in return, 40 bushels of wheat for 89 dollars 55 cts; 20 bushels of corn for £4 10s. 11d.; how much was there still to pay—expressed in decimal currency?

15. A man having 65 c. 95 ft. 123 in. of wood in his shed, sold 16 c. 117 ft. 65 in.; how much had he left?

16. A ship sailed on a whaling voyage, Aug. 25th, 1840, and returned April 15th, 1844; how long was she gone?

17. An apothecary had 9 lb 8 oz. 2 dr. 1 scr. 13 grs. of jalap, but has used in various mixtures 4 lb. 7 oz. 5 dr. 2 scr. 17 grs.; what has he left?

18. A note bearing date Oct. 20, 1823, was paid April 25, 1825; how many days was the note at interest?

MENTAL EXERCISES.

EXERCISE 29.

1. A bought of B a bushel of wheat for 7s. 6d. He gave him 1 bushel of corn worth 5s. 3d. and paid the rest in money. How much money did he pay?

2. A man sold a box of butter for 17s. 4d., and in pay received 7lb. of sugar, worth 9½d. per lb., and the rest in money. How much money did he receive?

3. A merchant bought a piece of cloth, containing 19 yds. 3 qrs., and sold 4 yds. 1 qr. of it; how much had he left?

4. A grocer drew out of a hhd. of wine 17 gals. 3 qts.; how much remained in the hogshead?

5. A smith bought 17 cwt. 3 qrs. of iron, and after having wrought a few days, wishing to know how much of it he had wrought, he weighed what he had left, and found he had 8 cwt. 1 qr. 13 lb. How much had he wrought?

6. C bought of B a bale of cotton for £18 4s., and B bought of C 4 barrels of flour for £9 3s. C paid B the rest in money. How much money did he pay?

7. When the minuend and the subtrahend are given, how do you find the remainder?

8. When the minuend and remainder are given, how do you find the subtrahend?

9. When the subtrahend and the remainder are given, how do you find the minuend?

10. When you have the *sum* of two numbers, and *one* of them given, how do you find the other?

11. When you have the *greater* of two numbers, and their *difference* given, how do you find the *less* number?

12. When you have the *less* of two numbers, and their *difference* given, how do you find the *greater* number?

13. When the sum and difference of two numbers are given, how do you find the two numbers?

NOTE.—The pupil may be required occasionally to give written answers to these mental exercises.

COMPOUND MULTIPLICATION.

Here again let us first do an example in *Simple*, and one in *Compound Multiplication* together, and compare the operation in each, in order to see that in Multiplication also, as in *Addition* and *Subtraction*, *Simple* and *Compound* are done in the same way—by the very same rule.

CASE I.—*When the multiplier does not exceed* 12.

EXAMPLE 1.

SIMPLE.

thds.	*hds.*	*tns.*	*units.*
7	8	6	4
			9
70	7	7	6

10)36 — 3–6	*Units to tens.* *Reduction.* Same as in Addition.
10)57 — 5–7	*Tens to hds.* *Reduction.* Same as in Addition.
10)77 — 7–7	*Hundreds to thousands.* Reduction. Same as in Addition.

EXAMPLE 2.

COMPOUND.

£	s.	d.
78	6	4¼
		9
704	17	4¼

4)18 — 4–2	*Farthings to pence.* *Reduction.* Same as in Addition.
12)40 — 3–4	*Pence to shillings.* *Reduction.* Same as in Addition.
2(0)5(7 — 2–17	*Shillings to pounds.* *Reduction.* Same as in *Addition.*

The first example, you perceive, is *Simple Multiplication*, the second example is *Compound*. But in both alike, after multiplying, we *reduce* the product to the *next higher*, just as we did in Addition, and by the very same rule: "*Divide* (the *sum* in Addition, the *product* in Multiplica-

tion) *by as many of the same as it takes to make one of the higher,"*—setting down the remainder and carrying the quotient to the next product, and so on—multiplying each figure, and reducing each product till all are done. Let the pupil himself do and compare examples 3rd and 4th.

EXAMPLE 3RD.—Multiply 98765 ×12.

EXAMPLE 4TH.—Multiply 7 mls. 5 fur. 26 per. × 12.

The rule then would be as follows :

RULE.—*When the multiplier does not exceed 12, beginning at the lowest, multiply each figure by the multiplier, and then reduce each product, as was directed to reduce each amount in Addition, setting down the remainder and carrying the quotient, as in Addition.*

EXERCISE 30.

(1)

cwt.	*qrs.*	*lbs.*
4	3	16
		4

(2)

lbs.	*oz.*	*dwts.*
24	3	12
		8

(3)

yds.	*qrs.*	*nls.*
16	3	2
		7

(4)

acs.	*rds.*	*per.*
3	2	27
		7

(5)

qrs.	*bu.*	*pks.*
7	6	2
		9

(6)

hrs.	*min.*	*sec.*
9	25	30
		6

7. Multiply 74 a. 2 r. 7 per. 4 yds. ×9.
8. Multiply £169 17s. 11¾d. ×9.
9. Multiply 111 cords 7 cu. ft. 7 cu. in. ×12.
10. Multiply 278 mls. 6 fur. 11 per. ×7.
11. Multiply 11 gal. 1 qt. 1 pt. ×11.
12. Multiply 169 cwt. 2 qrs. 11 lbs. ×6.

CASE II.—*When the multiplier exceeds* 12.

EXAMPLE.—Multiply 4625 × 36.

In this example we wish to get 36 times 4625. Now we may do this in two ways. For 6 times added to 30 times, would make 36 times: or, 4 times multiplied by 9 times, would make 36 times. See case 2d of Simple Multiplication.

FIRST WAY.

```
 4625
   36
------
 27750= 6 times.
13875 =30 times.
------
166500=36 times.
```

Simple Multiplication is usually done in the *first* way, as in this first example, (that is, by separating the multiplier into its parts, then finding each part separately, and adding the results together,) though it may be done in either way. Here we first get 6 times, then we get 30 times, and add the results.

SECOND WAY.

```
  4625
     4
------
 18500= 4 times.
     9
------
166500=36 times.
```

Compound Multiplication is usually done in the *second* way, as in this second example—that is, by resolving the multiplier into its *factors*, then multiplying by the *first* factor, and that product by the *second*, and so on; and when necessary, it is done by both these methods combined. The factors we take for 36 are 4 × 9. We first get 4 times, then multiplying 4 times by 9 we get 36 times.

The only new thing to be learned here, then, you observe, is what factors to take for any number. In every other respect the work is the same as when the multiplier does not exceed 12.

EXAMPLE.—Multiply 14 cwt. 3 qrs. 6 lbs. 6 oz. × 432= (10 × 10 × 4 + 32.)

tns.	cwt.	qrs.	lbs.	oz.	
	14	3	7	6	$\times 2 = 2$ times.
				10	
7	8	0	17	12	$\times 3 = 30$ times.
				10	
74	0	6	9	8	
				4	
296	6	1	10	0	= 400 times.
22	4	1	25	4	= 30 times.
1	9	2	14	12	= 2 times.
320	0	1	12	0	= to 432 times.

First we find the highest number of times—that is, the 400, for which the factors are $(10 \times 10 \times 4) = 400$ times. We first multiply by these factors to get the 400 times. Then we get the next highest, viz: the 30 times, in the most convenient way we can. The factors for the 30 are $(10 \times 3) = 30$, but we need not multiply by 10 to get the ten times, for we have in the second line 10 times already; therefore, we multiply the second line by 3 and get 30 times at once. We have now, yet, to get it, viz: the multiplicand (twice). This is got by multiplying the first line, (*i. e.*, the multiplicand) by 2. Now we have the result of repeating the multiplicand—first, 400 times; second, 30 times; and third, 2 times. These several results we add together, and get 432 times—the result we want.

The Rule then would be as follows :—

Rule for Compound Multiplication.—*When the multiplier exceeds 12, resolve the multiplier into its factors, then multiply the multiplicand by one factor, and the resulting product by another factor, and so on, till all the factors have been used. The last product will be the product required.*

If the multiplier cannot be directly resolved into factors, get the factors that will produce the highest number of times in it, and having multiplied by these, get the products of each of the remaining parts of the multiplier (always getting the highest first, and so proceeding to the lowest) in the most convenient manner possible, by using the products already obtained; then add the several results together.

$36 = 6 \times 6$; $100 = 10 \times 10$; $10000 = 10 \times 10 \times 10 \times 10$, &c.

EXAMPLE.—Suppose the multiplier to be $7852\frac{3}{4}$.

$7852\frac{3}{4} =$ {
1st part 7000, $=10\times10\times10\times7=7000$ times.
2d part 800, $=100$ (which we have already in our second product,) $\times8=800$ times.
3d part 50, $=10$ (which we have already in first product) $\times5=50$ times.
4th part 2, = the multiplicand multiplied $\times2=2$ times.
5th part $\frac{3}{4}$, = the multiplicand multiplied $\times\frac{3}{4}=\frac{3}{4}$ times.

These five results added together $= 7852\frac{3}{4}$ times.

EXERCISE 31.

1. In the same manner as in the above example, write out the factors for the following numbers, viz: $2876\frac{5}{8}$, 76527, $285876\frac{5}{7}$, $6754\frac{11}{12}$.

2. Multiply 17 lbs. 7 oz. 14 dwts. by 478.

3. Multiply £476 15s. 8d by 647.

4. Multiply 4 mls. 6 fur. 20 per. by 7426.

5. Multiply 36 yds. 2 qrs. 3 nls. by $8047\frac{3}{4}$.

6. Multiply 42 acres 1 rd. 10 per. by $576\frac{5}{9}$.

7. Multiply 27 qrs. 7 bush. 3 pks. by $807\frac{5}{6}$.

8. Multiply 32 yrs. 3 wks. 4 days by $236\frac{11}{12}$.

9. Multiply 4 cir. 7 signs 14 deg. by $723\frac{3}{4}$.

10. Multiply 7 reams 15 qrs. 9 sheets by $36\frac{7}{15}$.

11. Multiply \$476.17 by $865\frac{7}{12}$.

12. Multiply \$750.25 by $736\frac{4}{9}$.

13. How much silver in 6 table spoons, each weighing 5 oz. 10 dwts.?

14. What is the weight of 36 hhds. of tobacco, each hhd. weighing 5 cwt. 3 qrs. 14 lbs. 13 oz.?

15. If one spoon weigh 3 oz. 5 dwt. 15 grs., what is the weight of 120 spoons?

16. If a railroad car goes 21 m. 2 fur. 10 r. per hour, how far will it go in 15 hours?

17. How much cloth will it take to make the clothes for a regiment of soldiers containing 1143 men, if each suit requires 7 yds. 3 qrs. 2 nls. 1 in.?

18. If a steamship in going round the world travel 211 m. 4 fur. 32 per. a day, how far will she go in 367 days?

19. In 27 barrels there was on an average in each, 29 gallons, 3 quarts, 1 pint; how much in all?

20. I can go to a certain town by the railway in nine hours, 25 minutes, and 30 seconds; it would take me, at least, five times as long to go by the stage coach; how long would the coach take?

21. How much water will be contained in 96 hogsheads, each containing 62 gal. 1 qt. 1 pt. 1 gi.?

22. I bought 375 bales of English goods for £9 11s. 6d. per bale, and sold them for 16,000 dollars; what did I gain?

23. How much molasses is contained in 25 hhd. each hhd. having 61 gal. 1 qt. 1 pt.?

24. How much wood in 12 piles, each containing 7 cords 5 cu. ft. 12 cu. in.?

25. A farmer has 18 lots, and each lot contains 41 a. 2 r. 11 p.; how many acres does he own?

MENTAL EXERCISES.

EXERCISE 32.

1. At 7s. 4d. per bush., what cost 18 bushs. of wheat?
2. What cost 32 lb. of coffee, at 1s. 8d. per lb.?
3. What is the weight of 5 casks of rasins, each cask weighing 2 cwt. 3 qrs. 25 lb.?
4. At 27 cents a nail, what is the price of 2 yds. 1 qr. 3 nls. of cloth?
5. A market-woman bought 4 quarts of strawberries for 29 cents, and sold them at 5 cents a pint; what did she gain?

COMPOUND DIVISION.

In doing any question in *Division*, no matter of what kind, the various steps in the process are 5 in number, which 5 steps or operations are repeated over and over again till the question is done. These steps are, 1st. *Get the quotient figure.* 2nd. *Multiply the divisor by this quotient figure.* 3rd. *Subtract the product from that part of the dividend taken.* 4th. *Reduce the remainder to the next lower denomination; and,* 5th. *Add the result to whatever number of that lower denomination there may be in the dividend.* Now these 5 steps must be taken in every question in division, whether *Short, Long, Simple,* or *Compound.* But it is in *Compound Long Division* alone that all the figures and work of each step are fully written down. In each of the others, either some of these operations are done mentally, or two or more of them are contracted into one, and therefore do not appear on the slate.

In *Simple Short Division*, for instance, as the divisor is less than 12, the operations Nos, 2 and 3, are performed mentally, and the operations, Nos. 4 and 5, are also performed mentally, by supposing the remainder as so many tens to be placed before the next figure of the dividend, (just what it would come to if Nos. 4 and 5 were performed in full,) see annexed examples.

In *Simple Long Division*, the divisor being more than 12, the operations Nos. 2 and 3, are put down in full, but being simple numbers, where 10 of the lower make one of the higher constantly, operations Nos. 4 and 5 are contracted into one, by bringing down the next figure of the dividend to the right of the remainder, thus *reducing* and *adding* at once.

In *Compound Short Division*, the divisor again being less than 12, the operations Nos. 2 and 3 are done mentally, and the operations Nos. 4 and 5 are either done mentally or aside from the question, to serve a temporary purpose, and be immediately rubbed off, and consequently as in Simple Short, the operations Nos. 2, 3, 4, and 5 do not appear on the slate.

The following example, done first by putting the work of each step fully down, (as is done in *Compound Long Division,*) and then by performing some of the operations mentally, or contracting two or more into one, (as is done in *Compound Short* and in *Simple Long and Short,*) will best illustrate these remarks and this whole subject of *Division.*

EXAMPLE.—Divide 4893 by 9

I.

OPERATION IN
COMPOUND LONG—No. 1.

9)4893(543$\frac{6}{9}$
45 No. 2.
—
3 No. 3.
10 No. 4.
—
30
9 No. 5.
—
39 New dividend
36 No. 2.
—
3 No. 3.
10 No. 4.
—
30
3 No. 5.
—
33 New dividend.
27 No. 2.
—
6 No. 3, &c., &c.

II.

SAME OPERATION IN
SIMPLE LONG—No. 1.

9)4893(543$\frac{6}{9}$
45 No. 2.
—
39 Nos. 3, 4, and 5.
36 No. 2.
—
33 Nos. 3, 4, 5.
27 No. 2.
—
6

III. Same operation in Compound Short.	IV. Same operation in Simple Short.
9)4893	9)4893
543$\frac{6}{9}$	543$\frac{6}{9}$

3	No. 3.	3	No. 3.
10	No. 4.	10	No. 4.
30		30	
9	No. 5.	3	No. 5.
39	new div'd.	33	new div'd

This example, as we see here, can be done in any way from the most expanded to the most contracted, but if the divisor were more than 12, the operations Nos. 2 and 3 could not conveniently be done in the mind, and it could therefore only be done in the first and second ways; and if the dividend were of different denominations, so that the remainder could not be reduced by multiplying constantly by 10, then we could not reduce and add by bringing down, or calling the remainder so many tens; we would consequently have to do it in the first way.

Rule for Compound Division.—*Proceed as in Simple Division, but as the remainder cannot be reduced and added to the next lower, by bringing down the next figure, multiply it by as many of the next lower as it takes to make one of the same, adding in the given number of the next lower. Divide the number thus obtained by the divisor, as before, and so on.*

1.—EXAMPLE.

COMPOUND SHORT.

```
cwt. qrs. lbs. oz.
9)17   3   14   3
 ──────────────────
   1   3   23  12 7/9
 ──────────────────
   8 cwt. 1st rem.
   4
 ──
  32 red. to qrs.
   3 given qrs. added.
 ──
  35 new dividend.
 ──
   8 qrs. 2d rem.
  25
 ──
 200 red. to lbs.
  14 given lbs. added.
 ──
 214 new dividend.
 ──
   7 lbs. 3d rem.
  16
 ──
 112 red. to ozs.
   3 given ozs. added.
 ──
 115 new dividend.
```

2.—EXAMPLE.

COMPOUND LONG.

```
    £    s.  d.
19)25   18   6(1  7  3¼ 17/19
   19
   ──
    6  1st rem. £
   20
   ──
  138 red. to s. and s. ad'd.
  133
   ──
    5 2nd rem. s.
   12
   ──
   66 red. to d. and d. ad'd.
   57
   ──
    9 3rd rem. d.
    4
   ──
   36 red. to f.
   19
   ──
   17
```

EXERCISE 33.

1. Divide 19 cwt. 3 qrs. 8 lbs. by 3.
2. Divide 18 lbs. 6 oz. 14 dwts. by 17.
3. Divide 16 per. 2 yds. 1 ft. by 9.
4. Divide 64 yds. 2 qrs. 3 nls. by 42.
5. Divide 36 acr. 3 rd. 27 per. by 31.
6. Divide 9 qrs. 7 bush. 3 pks. by 9.
7. 127 cwt. 2 qrs. 17 lbs. ÷ 11.

8. £96749 16s. $11\frac{1}{4}$d ÷ 117.
9. 69 gals. 1 qt. 1 pt. ÷ 12.
10. 679 sq. per. 7 ft. 107 in. ÷ 132.
11. 290 sq. per. 7 yds. 8 ft. ÷ 3.
12. 1467 French ells 1 qr. 2 nls. 1 in. ÷ 267.
13. 172 days 16 h. 29 m. ÷ 7.
14. 916 miles 6 fur. 4 yds. ÷ 67.

We have now seen that with respect to the *operation*, Simple and Compound Division, and Division of every other name, may be said to be alike, and so with the other rules. The real distinction between questions in Division is not in the *mode of doing* them, but in the *nature of the question*. For instance,—if 1 barrel of flour cost $5, 20 barrels will cost $100. Here are 3 things,—1st, P. O. (*price of one*) $5. 2nd, P. W. (*price of the whole*) $100,—and 3rd, N. T. (*number of things*) 20 barrels.

Now, out of these 3 things we get 3 *distinct kinds of questions* :—

1st. If P. W. (price of the whole) be *required*, then, P. O. and N. T. must be given, and the question reads thus,—If 1 barrel of flour cost $5, what will 20 barrels cost? To do this, of course, *we repeat the price of one, once* for every thing or barrel, $5 × 20 = $100,—MULTIPLICATION. The multiplier is always an abstract number,—the product is always of the same name as the multiplicand.

2nd. If P. O. (price of one) be *required*, then P. W. and N. T. must be given; and the question now reads thus: If I pay $100 for 20 barrels of flour, what is the *price per barrel?* To do this of course we *divide the price* of the *whole into as many equal parts as there are things*, i. e. $100 ÷ 20 = $5. Here we are dividing a *thing* into a certain number of equal parts; the divisor is therefore an *abstract number*; and the *quotient* is of the *same name* as the *dividend*. This is DIVISION INTO EQUAL PARTS.

3rd. If N. T. (number of things) be *required*, then P. O. and P. W. must be given, and the question now reads: If I pay $100 for flour at $5 a barrel, *how many* barrels do I receive? To do this, we *find how often the price*

of one is contained in the price of the whole, and for every time it is contained in it, there will be one thing, (barrel, or whatever it may be), i. e. \$100 ÷ \$5 = 20 times. In this case, the quotient is always an *abstract number*, and the *divisor* and *dividend* must be of the same name. This is DIVISION, OR FINDING HOW OFTEN ONE THING IS CONTAINED IN ANOTHER OF THE SAME NAME.

Here, then, are two cases in Division quite distinct in their very *nature*, and every question in Division must be like the one or the other of these examples. Observe, in the one P. O. is *required*, and P. W. and N. T. *are given*. In the other N. T. is *required*, and P. O. and P. W. *are given*.

EXERCISE 34. (MENTAL.)

1. To find the *price of one*, what must be given in the question? Give an example, and state what is *given*, and what is to be found; and how you would proceed in doing it, then do it. Do the same with each of your following examples.

2. To find the *number of things* bought or sold, what must be given? Give an example, and do with it as before.

3. Give examples like the last in which the *price of one* and the *price of the whole* are of various denominations, and show *why*, in doing them, you have first to reduce the divisor and the dividend to the lowest name mentioned in either, then do the examples.

4. To find the *price of the whole*, what must be *given*? Give an example, and proceed with it as before.

5. Point out what three things are here, any two of which may be given to the find the third.

6. Show that this gives rise to *three distinct kinds* of questions, and give an example of each, stating of what kind each example is, and how it should be done.

7. What is the *name* or *kind*, of the *divisor* and of the *quotient*, in each kind of division, and of the *multiplier* and *product* in multiplication.

8. Give an example in which the price of 3 is given to find the price of 7. First state how you would proceed in doing it step by step, then do it accordingly.

9. What could you substitute for the word *price* in the three different kinds of questions you have been examining?

10. Give examples in which the price or weight, &c., first of $\frac{1}{2}$; second, of $\frac{3}{4}$; and third, of $5\frac{1}{2}$, is given, to find the price or weight, &c., of [illegible]7. First, state how you would proceed in doing each, then do each accordingly.

11. Give an example in which the price of $3\frac{1}{2}$, and the price of the whole are given to find the number of things. Proceed in explaining, and doing it as before.

12. Give examples of questions in which the *weight of the whole* and the weight of $\frac{3}{8}$, and of $4\frac{3}{4}$, and of $9\frac{1}{5}$, are given to find *the number of things*. Go through your plan of doing each, first, orally, and then do each as before.

NOTE.—Go through the following and the miscellaneous exercises, in the same manner before doing them. First, stating what is given and what required, and then, how you intend to proceed in doing it, giving each step in its proper order, then work it accordingly.

EXERCISE 35. (SLATE.)

1. If 36 bags of cotton weighed [illegible] cwt. [illegible] qrs. [illegible] lbs., how much did one weigh?

2. A gentleman sent a silver tankard to a silversmith, and ordered him to make it into spoons, each to weigh [illegible] oz. 12 dwt.; how many spoons did he make, the tankard weighing 4 lbs 7 oz.?

3. From Dublin to Cork is about 130 miles; how often does a coach wheel turn round between the two places, the circumference of the wheel being 12 feet?

4. How many hogsheads of sugar, each containing 13 cwt. 2 qrs. 14lbs. may be put on board a ship of 324 tons burden?

5. A silversmith received 36 lbs. 8 oz. 14 dwt. 16 grs. of silver to make 12 tankards; what would the weight of each tankard be?

6. If in 30 days a man travels 746 ml. 5 fur., travelling the same distance each day, what is the length of each day's journey?

7. A piece of cloth at 7s. 6d per yard, cost £17 12s. 6d.; how many yards were in it?

8. Bought 96 acres, 3 roods, 17 perches of land, for which I pay £1764; what did I pay for it per perch?

9. The area of a tract of bush country is 18233 acres 25 p. of land; and it is to be divided into lots, containing each 76 a. 2 r. 25p., of how many lots will it consist?

10. Bought 65 yards of cloth for which I paid £72 14s. 4½d.: what did it cost per yard?

11. A rich man divided 168 bu. 1 pk. 6 qt. of corn among 36 poor men; how much did each receive?

12. If a steamboat go 224 miles in a day, how long will it take to go to China, the distance being about 12,000 miles?

13. At $302.40 per tun, what will 1 hhd. 15 gal. 3 qts. of wine cost?

14. At $94.50 for 1 hhd. 15 gals. 3 qts. of wine, what is that per tun?

15. At $302.40 per tun, how much wine may be bought for $94.50.

16. At $2.215 per gals., what cost 3¼ qts.?

17. At $1.80 for 3¼ qts. of wine, what is that per gal?

18. At $2.215 per gal., how much wine may be bought for $1.80.

EXERCISE 36. (ON PRECEDING RULES.)

It is here earnestly recommended that, when going over this, and subsequent exercises the *first time*, each pupil, in turn, be required to state distinctly, in general terms, in the hearing of the class, before attempting to do any question—1st. What is *given* and *what is required* in each problem, as it comes before the class. 2nd. How

it is proposed to do it, giving each step neatly and briefly in its proper order.

If properly conducted, this will prove to be a most valuable training, not in *Arithmethic merely*, but in *Public Speaking*, in *Grammar*, *Composition*, and *Logic*.

If a pupil be thoroughly subjected to this training, day after day, at the black board, clearing up every problem, however complex, before the teacher and class, his success in all his other studies and undertakings is in a great measure determined and certain.

The class should be taken over the exercises a *second time* to test the relative quickness and accuracy of the pupils in the various operations.

EXERCISE 36.

1. Multiply 7896.43 by 99.9998, and divide the product by 786.597.

2. Express the following numbers in Roman Notation: 592, 4709, 69534.

3. Read the following numbers: 59.10967910, 113000-367189.104257, 89754621936, 10000001000.0001000.

4. Reduce $5694.25 to old Canadian currency.

5. A dealer bought 16 oxen in Canada, for £135 7s., and sold them in New York for $567.40; what did he gain by the transaction?

6. Multiply 749 lbs. 10 oz. (avoirdupois) by 725.

7. Suppose a man had 98 lbs. 2 oz. 19 dwt. 5 gr. of silver; how much must he give to each of 723 men if he divide it equally among them?

8. A man, on being asked his age, said he had spent the first 19 years of his life in England, the next 9 in America; during 27 following, 6 yrs. 11 months, 4 days, 6 wks. were spent in France, 16 yrs. 4 m. 3 days in the United States, and the remainder in his native country; how old was he, and in which land had he lived the longest?

9. 4 bags contain together $798.50; in the first there are $356.25, in the second $275.59, and in the third and fourth an equal amount; what sum *Canadian old currency* is there in each?

10. If I buy 12 boxes of oranges in New Orleans, at $5 per box, and by shipping them to Quebec sell them at 67s. per box, what would be my gain (in decimal currency) the freight being 6 dollars, and the waste from decay $\frac{1}{8}$ of the prime cost?

11. Find the exact length of the lunar month, which contains 2551443 seconds.

12. A pint will contain 9000 barley corns, and 5 of these placed one after another, would reach an inch, how far would they all reach?

13. Write down six hundred and twenty-nine billion, 97 million seven hundred and seven thousand four hundred and ninety-three, and six hundred and seventeen billionths.

14. Reduce 95679.367 inches to acres, roods, &c.

15. Take the number 67.95000000, and (by removing the dec. pt.); (1) multiply it by 1,000,000; (2) divide it by 10,000; (3); make it billionths; (4) make it millionths; (5) make it thousandths; (6) make it tenths.

16. If you buy 27 lbs. sugar, at 7d. a pound; 36 drums of figs, at 4s. $6\frac{1}{2}$d. a drum; 17 boxes of rasins, at 6s. 7d. a box; what will be the amount of your bill, in decimal currency?

17. A merchant having purchased 12 cwt. of sugar, sold at one time 3 cwt. 2 qr. 11lbs, and at another time he sold 4 cwt. 1 qr. 15 lbs.; what is the remainder worth at 15 cts per lb.?

18. The circumference of the earth contains one billion, five hundred and eighty-four millions of inches; express the same in miles.

19. In 19 fields there are 113 a. 3 r. 25 p. of land; if the fields contain an equal amount, how much is there in each field?

20. How many yards of carpet, 25 inches wide, will it take to cover a room 19 ft. 17 inches long, by 18 ft. 9 in. wide?

21. Divide $462 among 5 men and 6 women, giving to each man thrice the share of a woman.

22. One hundred and seventy-six men consumed in a week 13 cwt. 3 qrs. 15 lbs. 6 oz. of bread; how much did each man consume? What was the cost of the whole at 4s. 6d. a qr?

23. If one man consumes in a week 7 lb. 12 oz. 3 drams of bread, how many men will consume 13 cwt. 2 qr. 15 lb. 6 oz. in the same time? How many will consume it in one day at the same rate?

24. A farmer has a granary containing 232 bushels 3 pecks 7 quarts of wheat, and he wishes to put it in 105 bags, how much must each bag contain?

25. Divide £4876 9s. 3d. by four factors, which produce 630, and from the successive remainders find the true remainder.

26. If 5 oz. of silk can be spun into a thread $2\frac{1}{2}$ fur. long, what weight of silk will supply a thread of 100 miles?

27. If 3 qrs. 16 lbs. of silk is sufficient for a thread of 100 miles in length, what length of a similar size will 5 oz. spin?

28. A corn factor buys 2 qrs. at 39s. per quarter, and 7 bushels at 6s. per bushel, at what price per bush. must the whole be sold, so as to gain $4.75.

29. A side of Lincoln's Inn Square, London, is 770 ft., and of Russell Square 670 ft., how many acres do they contain respectively? What is the land worth at $460 per perch?

30. If 8 horses consume two-thirds of 889 bu. 2 pks. 6 qts. of oats in 365 days, what will one horse consume in one day?

31. If one horse consume in $\frac{2}{3}$ of a day 1 pk. 1 qt. 1 pt. 2 gill of oats; in how many days would 8 horses consume 889 bu. 2 pks. 6 qts.

32. If 7 horses consume in ¼ of a day, 1 pk. 1 qt. 1 pt. 2 gill of oats; how much would 8 horses cnnsume in 365 days?

33. A labourer dug a cellar 62 feet long, 25 feet wide, and 8½ ft. deep, at 5½d. per cubic yard; what was the amount of his bill in decimal currency?

34. In one pile of wood are 37 cords 119 cu. ft. 1462 cu. in.; in another, 9 cords 104 cu. ft.; in a third, 48 cords 7 cu. ft. 127 cu. in.; in a fourth, 61 cords 139 cu. in.; how much wood in the four piles, and its value at $3.84 a cord?

35. Bought 50 casks of molasses, each containing 58 gals. 3 qts., at 50 cts. per gal.; afterwards 215 gals. 2 qts. leaked out, and the remainder was sold at 3s. 4d. per gal.; what was the result of the operation?

36. Bought a piece of land 68 rods long, and 25½ rods wide, at £6 4s. 6d. per acre; what did it amount to?

37. A man lends his neighbour £135 6s. 8d., and takes in part payment 4 cows at $22 a piece, also a horse worth £50; how much remained due?

38. How many suits of clothes can be made from 3 pieces of cloth, each containing 39 yds. 2 qrs. 3 nls. every 4½ suits requiring 13 yds. 1 qr. 2 nls.

39. What weight of water may be contained in a canal whose depth is 8 ft., width 25 ft. and length 25 miles, when a cubic foot of water weighs 1,000 oz.

40. To how many persons may £60 15s. 6d. be distributed, giving £4 13s. 6d. to each?

41. Reduce 179 lbs. 3 oz. 3 dr. 1 scruple 14 grs. to grains.

42. Divide £1694 16s. 9d. by £9 19s. 11¾d.

43. Multiply 6 weeks 4 days 3 hrs. 17 min. by 429.

44. How many times would a coach wheel turn in a distance of 52 miles, the circumference of the wheel being 16 ft. 6 in.

45. A plate of gold cost $643.50 at $17.50 per oz.; what was its weight?

46. There is a certain number, to which if 4 be added, and from the sum 7 be subtracted, and the difference be multiplied by 8, and the product divided by 3, the quotient will be 64; what is that number.

47. Reduce 54622.716 cub. in. of firewood to cords.

48. A printer uses one sheet of paper for every 16 pages of an octavo book; how much paper will be necessary to print 500 copies of a book containing 336 pages, allowing 2 quires of waste paper in each ream?

49. If a ton of pot-ashes cost £27 10s., what will 14 cwt. cost?

50. If 14 cwt. of pot-ashes cost £19 5s., what is that per ton?

51. At £27 10s. a ton for pot-ashes, what quantity may be bought for £19 5s.?

52. If a bushel of wheat cost $1.92, what will 1 pt. 4 qts. cost?

53. If 1 pk. 4 qts. of wheat cost 72 cts., what is that per bushel?

54. At $1.92 per bushel, how much wheat may be bought for 72 cts.?

569597
481304
910567
814312
916727
&c., &c.

Let the pupil set down on the slate a column of 20 or 30 numbers, for addition, each number containing 5 or 6 figures, and let him *read* up their increasing sum to the top of the column, and then down, to see that he is correct, thus: beginning, (see first column), 7, 9, 16, 20, 27, and so on, until he can do so with the utmost *rapidity* and *correctness*.

55. In a written exercise, classify the questions in this 36th exercise into their different kinds by placing the No. of each question under its respective heading, and give

the rule accurately for doing each different kind of question. For example; thus,

DIVISION, (*Equal Parts*) Nos. 7, 22, &c.
DIVISION, (*How Often*) Nos. 20, 23, &c.
REDUCTION, (*Ascending*) Nos. 11, 18, &c.
REDUCTION, *Descending*) 41, &c.

EXERCISE 37. (MENTAL.)

1. To 5 add 9; subtract 3, add 1, take half of it, multiplying by 9, subtract 6, divide by 8, add 10; what is the result?

2. In £37 4s. how many shillings? How many dollars and cents?

3. What is the weight of 2 pockets of hops, each pocket weighing 1 cwt. 2 qrs. 17 lbs.?

4. How many yards of cloth that is 1 qr. wide, are equal to 27 yards, that is 1 yd. wide?

5. If 80 dollars will pay for 4 dinners for 30 men, how many dinners would it buy for 1 man?

6. How many coats may be made of 7 yds. of broadcloth, allowing 1 yd. 3 qrs. to a coat?

7. A farmer having 120 sheep, lost $\frac{1}{4}$ of them and sold $\frac{1}{2}$ of the remainder; he then bought 10 more; how many sheep did he then have?

8. What is the weight of 16 pigs of lead, each pig weighing 3 cwt. 2 qrs. 17 lbs.?

9. If an ounce of silver cost 6s. 9d., what is that per lb. Troy? What would 2 lb. 7 oz. cost?

10. If flour cost $5 a barrel, how many barrels can be bought with 6 barrels of pork, at $18 a barrel?

11. A person sold 2 bushels and 1 peck of currants, at 2 cents a pint, and in payment received 1 bushel of gooseberries, at 4 cents a pint; how much remains due?

12. At 9 dollars a barrel, how many barrels of beef may be bought for 3,827 dollars?

13. From a piece of cloth, which contained 43 yds. 1 qr., a tailor cut 3 suits, containing 6 yds. 2 qrs. 2 nls. each. How much cloth was there left?

14. At 17 cents per lb., how many pounds of chocolate may be bought for $5021? How many lbs. for $24?

15. A man bought a horse for 80 dollars, paid 2 dollars a week for his keeping, and received 4 dollars a week for his work;—at the expiration of 10 weeks he sold him for 70 dollars; how much did he gain by the operation?

16. A fox is 80 rods before a hound, and the hound gains 5 rods on the fox every 10 minutes; in how many minutes will the fox be caught? In how many hours?

17. Seven men bought a quantity of land for 84 dollars, and sold it for 14 dollars less than they gave for it; what did they sell it for, and what was each man's share of the loss?

18. A farmer sold 18 bushels of wheat at $2 a bushel, and took in part payment a calf at $5, and the balance in flour at $4 a barrel; how many barrels of flour did he receive?

19. A man bought a cow for £13½ and sold her for $15¾; he laid out what be gained in oats at 20 cents a bushel; how many bushels did he buy?

20. A man sold some wood for £2 8s., and received in pay 3 gals. of molasses at 2s. 3d. per gal., 8 lb. of raisins at 10d. per lb., 1 gal. of wine at 11s. 3d., and the rest in money. How much money did he receive?

21. If 6 bushels of wheat cost $7.50, what will 28 bushels cost?

22. If $\frac{3}{4}$ of a bushel cost $\frac{1}{3}$ of a $, what will 8 bushels cost?

23. If 16 men can finish a piece of work in 12 days, in what time can 20 finish it?

24. If 5 persons spend $\frac{5}{8}$ of a $, how much will 8 persons spend?

SECTION III.

GREATEST COMMON MEASURE; LEAST COMMON MULTIPLE; VULGAR FRACTIONS; AND DECIMAL FRACTIONS.

GREATEST COMMON MEASURE.

A *measure* or *factor* of a number is any number which divides it without a remainder. Thus, 2, 3, 4, 6, and 12, are all measures of 12. Any number that will divide each of two or more numbers without a remainder is called a *common measure* or factor, and the greatest number that will do this is called the *greatest common measure*, represented by the letters G. C. M. Thus 2 is a C. M. of 4 and 6; and 15 is the G. C. M. of 30 and 45.

To find the G. C. M. of two numbers :—

RULE.—*Divide the greater by the less and the last divisor by the last remainder, and so on, dividing the last divisor by the last remainder till nothing remains. The last divisor will be the G. C. M.*

EXAMPLE.—What is the G. C. M. of 132 and 312?

```
132)312(2
    264
    ---
     48)132(2
         96
         --
         36)48(1
            36
            --
            12)36(3
               36
               --
```

Here we divide the greater, 312 by the less, 132, and obtain the remainder 48. by which we now divide 132, the last divisor, and get the remainder, 36; and so on. The last divisor 12, is therefore the G. C. M.

EXERCISE 38.

Find the G. C. M, of the following numbers:

1. 224 and 336.
2. 175 and 2040.
3. 2121 and 1313.
4. 377 and 1131.
5. 900 and 3474.
6. 348 and 1024.
7. 1225 and 625.
8. 429 and 715.
9. 2431 and 770.
10. 1379 and 2401.

LEAST COMMON MULTIPLE.

A *multiple* of a number, is any number that will contain it without a remainder. Thus, 12 is a multiple of 1, 2, 3, 4, 6, and 12.

A *common multiple* of two or more numbers is, any number that will contain each of them without a remainder; and the least number that will do this is called the *least common multiple* represented by the letters *l. c. m.* Thus, 16 is a c. m. of 1, 2, 4, and 8, but 8 is the *l. c. m.* of these numbers.

To find the *l. c. m.* of two or more numbers :—

RULE.—*Write the given numbers in a line, and strike out any that are contained in any of the others without a remainder. Set aside any one of the numbers not struck out, as a factor to be retained, and divide each of the remaining numbers of the line by the greatest number that will exactly divide it and the number set aside, bringing down the quotients and any numbers that cannot be thus divided for the next line.*

Proceed with this line as with the first line, and so on with each succeeding line until no number will exactly divide any two numbers in the line.

Then, multiply all the factors set aside, and all the numbers left in the last line together, and the product will be the l. c. m.

EXAMPLE.—What is the *l. c. m.* of 4, 6, 8, 10, 12, 16, 20, 24, 25, and 30?

OPERATION.

16)~~4~~ ·· ~~6~~ ·· ~~8~~ ·· ~~10~~ ·· ~~12~~ ·· 16 ·· 20 ·· 24 ·· 25 ·· 30

15) ~~5~~ ·· ~~3~~ ·· 25 ·· 15

5

16×15×5=1200, *l. c. m.*

Here 4, 6, 8, 10, and 12, are struck out at once as each of them is contained in some other of the given numbers. We then set aside 16 as a factor to be retained, and then divide 20 by 4, because 4 is the greatest number that divides 16 and 20 without a remainder, for the same reason we divide 24 by 8 and 30 by 2, and as there is no factor common to 16 and 25, we bring down the 25 Now we have in the new line 5, 3, 25, and 15; we strike out 5 and 3, set aside 15, and divide 25 by 5. We then have 16, 15, and 5, as the factors left, which give the product 1200, the *l. c. m.*

Find the *l. c. m.* of:

1. 15, 20.
2. 8, 4, 16.
3. 12, 15, 16.
4. 9, 15, 18, 20.
5. 8, 12, 15, 20.
6. 6, 12, 16, 18, 24.
7. 2, 4, 8, 16, 10, 48.
8. 7, 12, 15, 27, 35, 40, 45.
9. 4, 9, 10, 15, 18, 20, 21.
10. 8, 9, 10, 12, 25, 32, 75, 80.

VULGAR FRACTIONS.

When the denominator is followed by one or more 0's, it is called a *Decimal Fraction.* For instance, $\frac{3}{10}$, $\frac{17}{100}$,

All other fractions are called Vulgar or Common Fractions, and they are divided into *Proper, Improper, Simple, Compound, Complex, and Mixed.*

A Proper Fraction is one that has its numerator less than the denominator, as $\frac{3}{5}$, $\frac{5}{9}$, $\frac{11}{12}$.

An Improper Fraction is one that has its numerator equal to, or greater than the denominator. As, $\frac{3}{3}$, $\frac{13}{6}$, $\frac{21}{12}$.

A Simple Fraction is a single expression, either proper or improper, expressing one or more equal parts of unity. Thus, $\frac{3}{4}$, $\frac{9}{9}$, $\frac{16}{15}$, are simple fractions.

A Compound Fraction is a fraction of a fraction, or several fractions connected by the word *of.* As, $\frac{1}{3}$ of $\frac{5}{9}$, $\frac{3}{5}$ of $\frac{11}{2}$, $\frac{2}{5}$ of 3 of $\frac{16}{9}$.

A Complex Fraction is one which has one or more of any of the other kinds of fractions for its numerator, or for its denominator, or for both. As, $\dfrac{\frac{3}{4}}{2\frac{5}{7}}$, $\dfrac{4\frac{2}{5}}{\frac{7}{9}}$, $\dfrac{\frac{2}{3}\text{ of }\frac{4}{5}}{\dfrac{2\frac{5}{7}}{\frac{3}{4}}}$.

A Mixed Number is one composed of a whole number and a fraction. As, $14\frac{1}{2}$, $13\frac{7}{8}$, &c.

If 15 apples were divided equally among 4 boys, how many would each boy receive?

If it were 12 instead of 15 apples, each boy would of course get exactly 3; and if it were 16 apples, each boy would receive exactly 4; but as there are 15 apples to be divided equally among 4, 3 is less than the exact share of each, and 4 is more, hence, each boy must get more than 3 and less than 4. After each of the 4 boys has received 3 apples there are still 3 remaining (not yet divided.)— These remaining 3 must also be divided into 4 equal parts, and one of these parts given to each boy in addition to the 3 whole apples he has already received.— That is, the exact share of each boy will be 3 apples and the $\frac{1}{4}$ part of 3, or (which is the same thing) 3 and the $\frac{3}{4}$ part of one. This $\frac{3}{4}$ is called a *Vulgar* or *Common Fraction.* The upper figure 3 is called the NUMERATOR, and the lower figure 4 is called the DENOMINATOR.

We learn from the examination of this question—

I. How fractions originate.

They arise from questions in Division, and are themselves questions in division, the numerator being the dividend and the denominator the divisor.

II. What a fraction is.

A fraction is an expression representing one or more of the equal parts into which any quantity may be divided.

III. What the *Denominator* represents.

The Denominator, (according to the manner in which it is viewed,) represents the divisor,—the name of the fraction,—the number of equal parts into which the quantity is divided,—the number it takes to make an integer or whole thing,—the size of the pieces.

IV. What the *Numerator* represents.

The Numerator represents the dividend, also the number of parts taken.

EXERCISE 40.

This Introductory exercise is intended to exhibit and apply the principles connected with the operations in fractions. The teacher is supposed to illustrate each principle, and question under it, and then exercise the class on it before proceeding to the rest.

1. How do fractions arise?—Give an example.

2. What is a fraction?—Give examples of each different kind. Write them out and read them.

3. What does the Denominator represent?

4. Show how the denominator represents the *size* of the pieces?

5. What does the Numerator represent?

I. Remember, then, *the Denominator represents the size of the pieces—and the Numerator represents the number of the pieces.* This being the case,

6. Compare $\frac{3}{4}$ and $\frac{3}{16}$, and show what effect it has had upon the *value* of the fraction $\frac{3}{4}$ to multiply the denominator by 4.

7. Compare $\frac{3}{12}$ and $\frac{3}{4}$, and show what effect it has had upon the *value* of the fraction $\frac{3}{12}$ to divide the denominator by 3.

8. Compare $\frac{2}{9}$ and $\frac{8}{9}$, and show what effect it has had upon the value of the fraction $\frac{2}{9}$ to multiply the numerator by 4.

9. Compare $\frac{8}{16}$ and $\frac{4}{16}$, and show what effect it has had, upon the value of the fraction $\frac{8}{16}$ to divide the numerator by 2.

How, then, may the value of a fraction be *increased* or *decreased* by *multiplying*, and how may the value be *increased* or *decreased* by *dividing?*

10. Compare $\frac{3}{4}$ with $\frac{3}{16}$, and $\frac{12}{16}$ with $\frac{4}{6}$, and show what effect it has had upon the value of the fraction $\frac{3}{4}$ to multiply

the numerator and denominator *by the same number*, and what effect upon the value of the fraction $\frac{12}{18}$ to divide the numerator and denominator *by the same number*.

II. Remember, that, *the denominator represents the number of pieces it takes to make a whole one, and numerator the whole number of pieces in the fraction*. This being the case,

11. How many whole ones are there in $\frac{4}{4}$, $\frac{8}{8}$, $\frac{12}{12}$? in $\frac{13}{8}$, $\frac{27}{8}$, $\frac{63}{8}$, $\frac{17}{9}$? in $\frac{9}{1}$, $\frac{8}{1}$, $\frac{27}{1}$?

This is reducing an improper fraction to a whole or mixed number. Write out a rule for doing it.

12. How many 6ths in 7, 12, 5, whole numbers? How many 12ths in 15, 30, 46?

This is reducing a whole number to a fraction having a given denominator. Write out a rule for doing it.

13. How many 6ths in $8\frac{2}{6}$, in $12\frac{5}{6}$, in $17\frac{3}{6}$? How many 12ths in $17\frac{11}{12}$, $15\frac{9}{12}$, $20\frac{7}{12}$?

This is reducing a mixed number to an improper fraction. Write out a rule for doing it.

III. Remember that *dividing the numerator and denominator of a fraction by the same number does not alter its value.* This being the case—

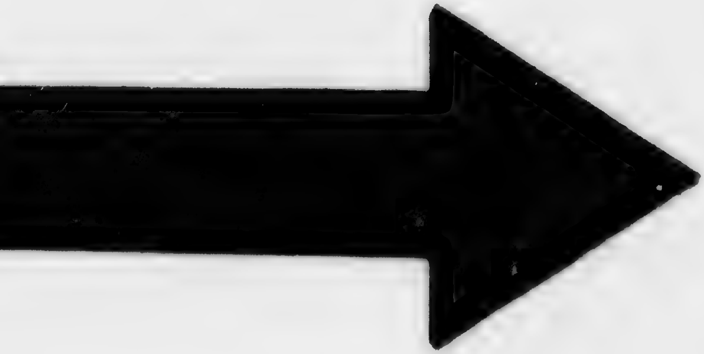
14. Give each of the following fractions the least numerator and denominator it can have, without altering the value, viz.: $\frac{3}{9}$, $\frac{6}{36}$, $\frac{10}{12}$, $\frac{11}{44}$, $\frac{12}{15}$, $\frac{11}{33}$, &c.

This is reducing a fraction to its lowest terms. Write out a rule for doing it.

IV. Remember that *multiplying the numerator and denominator of a fraction by the same number, does not alter its value.* This being the case—

15. Show how, according to this principle, each four of the following fractions may all be made to have the same denominator without altering their value, viz.: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{6}$, and $\frac{4}{5}$, $\frac{3}{5}$, $\frac{9}{10}$, $\frac{10}{15}$, and $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{12}$, $\frac{7}{20}$, and $\frac{11}{15}$, $\frac{7}{10}$, $\frac{9}{20}$, $\frac{7}{45}$.

[redacted]



This is reducing fractions to a common denominator, (which must be done before fractions having different denominators can be either added or subtracted). Write out a rule for doing it, and also for reducing them to the *least common denominator.* (Teacher explain to the class on the black-board.)

V. Remember that either *multiplying the numerator or dividing the denominator, increases the value of the fraction,* i. e., *multiplies the fraction; and that multiplying the denominator or dividing the numerator, lessens the value of the fraction,* i. e., *divides the fraction; and that any whole number may* (without altering its value,) *be written as a fraction, by placing 1 under it.* This being the case—

16. Multiply, and then divide, $\frac{5}{8}$ by 6, $\frac{16}{24}$ by 3, $\frac{12}{16}$ by 4; 18 by $\frac{3}{8}$, 12 by $\frac{3}{4}$, 40 by $\frac{4}{15}$; $\frac{18}{36}$ by $\frac{3}{8}$—(that is, get $\frac{3}{8}$ of $\frac{18}{36}$,) $\frac{27}{36}$ by $\frac{3}{8}$, $\frac{12}{32}$ by $\frac{4}{1}$.

This is multiplying and dividing (I.) *a fraction by a whole number,* (II.) *a whole number by a fraction,* (III.) *a fraction by a fraction.* Write out a separate rule for doing each, (that is, six rules), and then embrace the three for multiplication in one rule, and the three for division in one rule.

VI. Remember that *the numerator of a fraction is the dividend, and the denominator is the divisor, and that the word "of" between fractions, is the same as* ($\times$ *into*) *the sign* of multiplication. This being the case—

17. Do the multiplication and division required in the following fractions, viz: $\dfrac{\frac{2}{3} \text{ of } 2\frac{1}{2} \text{ of } 7}{\frac{5}{8} \text{ of } 2 \text{ of } 1\frac{3}{4}}$, $\dfrac{\frac{3}{4}}{\frac{9}{10}}$ of $\dfrac{\frac{1}{3}}{\frac{3}{4}}$ of $\dfrac{\frac{6}{7}}{\frac{3}{8}}$, $\dfrac{2\frac{1}{4}}{1\frac{1}{2}}$ of 2, of $\dfrac{1\frac{1}{3}}{2\frac{1}{2}}$ of 6.

This is reducing compound fractions, and complex fractions, to simple fractions. Write out a rule for doing each.

VII. Remember that the *numerator is the number of the things or pieces, and that the denominator shows their name, and that things of different names cannot be added together,*

nor subtracted, the one from the other. This being the case—

18. Add the following fractions, $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$, $\frac{5}{9}+\frac{5}{12}+\frac{9}{24}$, $\frac{5}{18}+\frac{8}{16}+\frac{4}{5}$, $\frac{15}{40}+\frac{7}{35}+\frac{8}{15}$.

19. Subtract $\frac{7}{8}-\frac{5}{6}$, $\frac{5}{12}-\frac{3}{32}$, $\frac{4}{9}-\frac{3}{35}$, $\frac{9}{12}-\frac{7}{30}$.

This is adding and subtracting fractions. Write out a rule for doing each.

VIII. Remember that *reduction descending is done by multiplying the fraction, and that reduction ascending is done by dividing the fraction.* This being the case—

20. Reduce $\frac{5}{8}$ of a lb to the fraction of a cwt.
   " $\frac{5}{7}$ of a year to months, weeks, &c.
   " $\frac{12}{13}$ of an hour to the fraction of a week.
   " $\frac{4}{11}$ of a £ to shillings, pence, &c.

What part of a £ is $\frac{5}{7}$ of a shilling?

What part of a month is $\frac{3}{4}$ of a day?

In the preceding Mental Exercise (40) the design has been to exhibit and apply all the principles involved in the various operations in fractions, so as to exercise the mind of the learner, and call forth the illustrations of the teacher. If this has been done to the extent designed, nothing further is necessary, except to collect the rules carefully worded, and furnish sufficient exercises for practice. The pupil should compare the rules he has already written, as directed, with the rules here given, and notice particularly in what respects they agree, and in what respects they differ.

RULES WITH EXERCISES.

To reduce an improper fraction to a whole or mixed number—

RULE.—*Divide the numerator by the denominator, the quotient will be the whole or mixed number.*

EXAMPLE. $\frac{19}{8}=2\frac{3}{8}$.

EXERCISE 41.

Reduce to their equivalent whole or mixed numbers:

(1) $\frac{313}{16}$, (2) $\frac{76}{9}$, (3) $\frac{63}{7}$, (4) $\frac{14}{11}$, (5) $\frac{52864}{6328}$, (6) $\frac{643}{128}$.

To reduce a mixed number to an improper fraction—

RULE.—*Multiply the whole number by the denominator, adding in the numerator, and under the result place the denominator.*

EXAMPLE. $6\frac{3}{5} = \frac{33}{5}$.

EXERCISE 42.

Reduce to improper fractions:

(1) $12\frac{3}{5}$, (2) $136\frac{11}{13}$, (3) $75\frac{6}{13}$, (4) $10\frac{3}{10}$, (5) $77\frac{7}{17}$, (6) $123\frac{6}{17}$.

To reduce a whole number to a fraction having a given denominator—

RULE.—*Multiply the whole number by the given denominator, and under the result place the given denominator.*

EXAMPLE. 12 to 6ths, $12 = \frac{72}{6}$.

EXERCISE 43.

Reduce (1) 8, (2) 7, (3) 12, (4) 106, (5) 72, (6) 908, each one to 9ths, 11ths, 17ths, 12ths, 36ths, 40ths.

To reduce a fraction to its lowest terms—

RULE.—*Divide the numerator and denominator by the G. C. M.; or divide the numerator and denominator by any number that will exactly divide them, and the reduced numerator and denominator again by any number that will exactly divide them, and so on, till no number greater than one will again exactly divide them.*

EXAMPLE. $\frac{12}{24} = \frac{1}{2}$, or $\frac{12}{24} = \frac{6}{12} = \frac{3}{6} = \frac{1}{2}$.

OBSERVE.—1st. Any number that ends in 5 is divisible by 5.

2nd. Any number that ends in 0 is divisible by 10, 5, or 2.

3rd. Any number that ends in an even number is divisible by 2.

4th. When the two right-hand figures are divisible by 4, the whole is divisible by 4.

5th. When the three right-hand figures are divisible by 8, the whole number is divisible by 8.

6th. When the sum of the digits of a number is divisible by 3, the number itself is divisible by 3.

EXERCISE 44.

(1) (2) (3) (4) (5) (6)

Reduce 1587/[illegible], 2817/[illegible], 3072/[illegible], 3909/[illegible], 8737/[illegible], 4794/[illegible] to their lowest terms.

To reduce a compound fraction to a simple fraction—

RULE.—*Write the whole numbers in the form of fractions, and reduce the mixed numbers to improper fractions, and the complex fractions to simple ones, then multiply all the numerators together for a numerator, and all the denominators together for a denominator.*

Complex fractions are reduced to simple fractions by *dividing the numerator reduced to a simple fraction by the denominator reduced to a simple fraction.* (See rule for Division.)

NOTE.—Before multiplying the numerators and denominators together, the work may be greatly shortened by cancelling all the factors that are common to any numerator and denominator.

EXAMPLE. $\frac{3}{9}$ of 4 of $6\frac{1}{12}$ of $\frac{1\frac{1}{2}}{\frac{1}{4}} = \frac{3}{9} \times \frac{4}{1} \times \frac{73}{12} \times \frac{12}{2} =$

$$\frac{\cancel{3}}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{4}}}{1} \times \frac{73}{\cancel{12}} \times \frac{\cancel{12}}{\cancel{2}} = \frac{146}{3} = 48\frac{2}{3}$$

EXERCISE 44.

(1) $\frac{5}{8}$ of $\frac{27}{40}$ of $\frac{10}{18}$ of $\frac{7}{8}$, (2) $6\frac{3}{4}$ of $\frac{9\frac{1}{2}}{8}$ of 2 of $\frac{5}{6}$, (3) $\frac{5\frac{1}{2}}{\frac{1}{3}}$ of 1 of $2\frac{1}{2}$ of $\frac{\frac{3}{4}}{2\frac{1}{4}}$,

(4) $\frac{5}{8}$ of $\frac{4\frac{1}{3}}{8}$ of $1\frac{5}{9}$, (5) $\frac{2\frac{1}{4}}{12}$ of 6 of $\frac{7}{8}$ of $5\frac{1}{3}$, (6) $\frac{1}{3}$ of $\frac{3}{8}$ of $1\frac{1}{2}$ of $\frac{\frac{3}{4}}{17\frac{3}{8}}$.

To add and subtract fractions.

RULE.—*Reduce them to a common denominator, then, for addition, add the numerators, and for subtraction, subtract the numerator of the subtrahend from the numerator of the minuend, and under the result place the common denominator.*

NOTE.—*At the close of every question in Addition, Subtraction, Multiplication, or Division, always reduce improper fractions to whole or mixed numbers, and proper fractions to their lowest terms.*

To reduce fractions to a common denominator.

RULE.—*Multiply the numerator and denominator of each fraction by all the denominators except its own.*

EXAMPLE.—Reduce to a common denominator and add $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.

OPERATION. $\left\{\begin{array}{l} \frac{3 \times 6 \times 8}{4 \times 6 \times 8} = \frac{144}{192} \\ \frac{5 \times 4 \times 8}{6 \times 4 \times 8} = \frac{160}{192} \\ \frac{7 \times 4 \times 6}{8 \times 4 \times 6} = \frac{168}{192} \end{array}\right\} = \frac{472}{192} = 2\frac{88}{192} = 2\frac{11}{24}$, result when added.

Or the following rule, which is generally a shorter method :—

To reduce fractions to their least common denominator.

RULE.—*Find the least common multiple of all the denominators, and take it for the common denominator. Then mul-*

tiply the numerator of each fraction by the quotient obtained by dividing its denominator into this common denominator.

EXAMPLE.—(With same fractions as above.)

$$\text{OPERATION.}\quad \left\{\begin{array}{l} \frac{3}{4} = \frac{18}{24} \\ \frac{5}{6} = \frac{20}{24} \\ \frac{7}{8} = \frac{21}{24} \end{array}\right\} = \frac{59}{24} = 2\frac{11}{24},\ \text{result when added.}$$

NOTE.—Before finding the l. c. m. by the rule, always see first if it cannot be found by *inspection*—that is, try whether the greatest of the given numbers is not it; if not, try twice the greatest, then three times, &c. Observe, in the last Example, is 8 it? No. Is 16? No. Is 24? Yes, 24 is the least number that will contain each of the denominators without a remainder.

EXAMPLE.—Subtract $\frac{2}{6}$ from $\frac{3}{4}$.

$$\text{OPERATION.}\quad \left\{\begin{array}{l} \text{From } \frac{3}{4} = \frac{9}{12} \\ \text{Take } \frac{2}{6} = \frac{4}{12} \\ \hline \qquad\qquad \frac{5}{12},\ \text{result.} \end{array}\right.$$

EXERCISE 45.

Reduce to a common denominator, then add and subtract the following fractions.

1. $\frac{3}{8} + \frac{2}{7} + \frac{5}{6} + \frac{3}{5} + \frac{1}{2}$.
2. $(\frac{7}{24} + \frac{11}{12} + \frac{3}{8}) - (\frac{7}{10} + \frac{13}{15}.)$
3. $(\frac{17}{20} + \frac{9}{10} + \frac{4}{15}) - (\frac{23}{36} + \frac{5}{8}.)$
4. $(\frac{7}{8} + \frac{5}{9} + \frac{3}{10}) - (\frac{11}{12} + \frac{3}{14}.)$
5. $(\frac{11}{12} + \frac{14}{17} + \frac{13}{15}) - \frac{17}{36}$.
6. $(\frac{13}{17} + \frac{9}{10} + \frac{15}{16}) - \frac{7}{12}$.

7. $\left(\frac{3}{10} + \frac{11}{12} + \frac{13}{18} + \frac{19}{21}\right) - \left(\frac{2}{3} + \frac{17}{30} + \frac{11}{20}\right)$.
8. $\left(\frac{2}{3} + \frac{4}{9} + \frac{5}{21}\right) - \left(\frac{4}{9} + \frac{1}{35}\right)$.
9. $\frac{17}{18} - \frac{8}{15}$, $\frac{74}{105} - \frac{32}{147}$, $\frac{276}{435} - \frac{148}{675}$.

To add mixed numbers.

Rule.—*First, add the fractions, and having obtained what they amount to, add it to the whole numbers.*

To subtract mixed numbers.

Proceed in the same way, first subtract the fractions, then the whole numbers.

EXAMPLE, ADDITION.

$$
\begin{array}{ll}
 & 8\tfrac{3}{4} = 8\tfrac{18}{24} \\
 & 17\tfrac{5}{6} = 17\tfrac{20}{24} \\
 & 23\tfrac{7}{8} = 23\tfrac{21}{24} \\
\hline
\text{Result} & 48\tfrac{59}{24} = 48 + 2\tfrac{11}{24} = 50\tfrac{11}{24}.
\end{array}
$$

EXAMPLE, SUBTRACTION.

$$
\begin{array}{ll}
\text{From} & 12\tfrac{3}{4} = 12\tfrac{6}{8} \\
\text{Take} & 9\tfrac{7}{8} = 9\tfrac{7}{8} \\
\hline
\text{Result} & 2\tfrac{7}{8}
\end{array}
$$

EXERCISE 46.

1. $(5\frac{1}{3} + 17\frac{3}{4}) - (6\frac{1}{2} + 1\frac{17}{18})$.
2. $(25\frac{21}{32} + 16\frac{2}{3} + 10\frac{1}{2}) - 13\frac{5}{6}$.
3. $(365\frac{7}{25} + 56\frac{1}{15} + 10\frac{7}{20}) - 133\frac{69}{785}$.
4. $(1287\frac{15}{48} + 715\frac{211}{750} + 8\frac{1}{3}) - (10\frac{1}{2} + 6\frac{7}{12} + 5\frac{1}{8})$.
5. $(10\frac{1}{3} + 9\frac{17}{18} + 63\frac{28}{35}) - (3\frac{1}{3} + 30\frac{3}{10} + 5\frac{69}{70})$.
6. $(725\frac{72}{235} + 18\frac{3}{4} + 13\frac{8}{25}) - (60\frac{1}{30} + 10\frac{1}{10} + 80\frac{1}{80})$.

To multiply fractions, or a fraction, of any kind by fractions, or by a fraction, of any kind.

Rule.—*Reduce compound and complex fractions to simple ones, and whole and mixed numbers to improper fractions, then, having cancelled the factors that are common to any numerator and denominator—*

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

EXAMPLE. $\frac{8}{12} \times \frac{3}{4} = \frac{\overset{2}{\cancel{8}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{12}}} \times \frac{\cancel{3}}{\cancel{4}} = \frac{1}{2}$

Or, $\frac{5}{9} \times 3 = \frac{5}{9} \times \frac{3}{1} = \frac{5}{\underset{3}{\cancel{9}}} \times \frac{\cancel{3}}{1} = \frac{5}{3} = 1\frac{2}{3}$

Or, $12 \times \frac{3}{4} = \frac{12}{1} \times \frac{3}{4} = \frac{\overset{3}{\cancel{12}}}{1} \times \frac{3}{\cancel{4}} = \frac{9}{1} = 9$

Or, $2\frac{3}{4} \times 2 \times \frac{6}{7} = \frac{11}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{1} \times \frac{\overset{3}{\cancel{6}}}{7} = 4\frac{5}{7}$

EXERCISE. [illegible] $\times$ [illegible] = what? [illegible] $\times 6$ = what? $36 \times$ [illegible] = what? [illegible] $\times$ [illegible] = what? $\frac{1\frac{1}{2}}{[illegible]} \times \frac{[illegible] \text{ of } [illegible]}{3\frac{1}{2}} \times 7$ = what?

To divide fractions, or a fraction, of any kind by fractions, or by a fraction, of any kind.

RULE.—*Reduce compound and complex fractions to simple ones, and whole and mixed numbers to improper fractions, as in Multiplication. Then,* INVERT THE TERMS OF THE DIVISOR, *and proceed as in multiplication.*

EXAMPLE. $\frac{8}{12} \div \frac{3}{4} = \frac{8}{12} \times \frac{4}{3} = \frac{8}{\underset{3}{\cancel{12}}} \times \frac{\cancel{4}}{3} = \frac{8}{9}$

Or, $\frac{5}{9} \div 3 = \frac{5}{9} \times \frac{1}{3} = \frac{5}{27}$.

Or, $12 \div \frac{3}{4} = \frac{12}{1} \times \frac{4}{3} = \frac{\overset{4}{\cancel{12}}}{1} \times \frac{4}{\cancel{3}} = \frac{16}{1} = 16$.

Or, $2\frac{3}{4} \div$ (2 + [illegible]) = [illegible] $\div$ ([illegible] + [illegible]) = [illegible] $\times$ [illegible] = [illegible] = [illegible].

EXERCISE. $\frac{3}{4} \div \frac{5}{6} =$ what? $\frac{5}{8} \div 6 =$ what? $36 \div \frac{6}{13}$ $=$ what? $\frac{176}{10} \div \frac{78}{85} =$ what? $\dfrac{7\frac{1}{2}}{6} \div \left(8 \div \dfrac{6}{9\frac{1}{3}}\right)$ $=$ what?

EXERCISE 47.

Find the value of:

1. $\frac{8}{11} \times (2\frac{1}{4} \times 3\frac{1}{2})$.
2. $\frac{8}{11} \div (2\frac{1}{4} \div 3\frac{1}{2}.)$
3. $3 \times 7\frac{1}{2} \times \frac{11}{13} \times 3\frac{8}{11}$.
4. $3 \div (7\frac{1}{2} \div 3\frac{8}{11})$.
5. $(11\frac{3}{4} + 6\frac{1}{5}) \times (9\frac{3}{5} + 7\frac{1}{7})$.
6. $(11\frac{3}{4} + 6\frac{1}{5}) \div (9\frac{3}{5} + 7\frac{1}{7})$.
7. $5\frac{3}{11}$ of $\dfrac{4\frac{1}{4}}{7\frac{3}{5}}$ of $\dfrac{1}{9\frac{1}{4}} \times \dfrac{8\frac{3}{5}}{9} \times 11\frac{1}{7}$.
8. $(\frac{1}{2} + \frac{3}{4} + \frac{4}{5} + \frac{6}{7}) \div \frac{2}{3}$ of $\frac{1}{2}$.
9. $9\frac{1}{4}$ of $\dfrac{8\frac{1}{2}}{\frac{7}{8}} \div 6\frac{2}{5}$ of $\dfrac{4\frac{3}{7}}{8\frac{1}{4}}$.
10. $(\frac{2}{3} + \frac{7}{8} + 8\frac{3}{4}) \div (\frac{6}{7} + 4\frac{3}{5} + \frac{19}{20}$.
11. $(8\frac{3}{11} + 2\frac{1}{7} + 3\frac{1}{10}) - (7\frac{2}{3} + 6\frac{5}{12} + 2\frac{7}{9} + 2 + 2\frac{1}{4})$ $\div (\frac{1}{2}$ of $11\frac{1}{2} + \frac{6}{7})$.
12. $\left(\dfrac{2\frac{1}{2}}{3\frac{1}{4} \text{ of } 2\frac{1}{3}} + \dfrac{6}{3\frac{1}{2}} + 17\right) \div \left(\dfrac{5\frac{1}{2}}{\frac{3}{14}} + \dfrac{2\frac{1}{3} + 17 + 8\frac{1}{3}}{2\frac{1}{3} + 8\frac{1}{3}}\right)$
13. $\frac{6}{7}$ of $\frac{4}{5}$ of $(\frac{1}{2} + \frac{3}{4}) \times \frac{8}{11}$ of $\frac{7}{18}$.
14. $\dfrac{4\frac{3}{4}}{7\frac{1}{2}} \div \left(\dfrac{6\frac{1}{2}}{\frac{1}{4}} \div \frac{1}{5} \text{ of } 3\frac{3}{7} \text{ of } 9\frac{2}{3}\right)$.
15. $\frac{7}{9}$ of $\frac{3}{5}$ of £1 16s. $8\frac{1}{2}$d.
16. $\frac{1}{3}$ of $\frac{2}{5}$ of $6\frac{3}{4}$ of $5\frac{3}{7}$ of $\frac{11}{13}$ of 6 lbs. 4 oz. Avoir.
17. $\frac{11}{16}$ of $\frac{3}{7}$ of $\frac{8}{33}$ of $6\frac{1}{4}$ acres.

18. $\frac{6}{7}$ of $\frac{4}{5}$ of $\frac{11}{12}$ of 3 acres 1 rood 27 per.

19. 4 acres 6 per. 5 yds. $\div 5\frac{1}{2} \times \frac{6}{7} \times \frac{18}{33}$.

20. 11 cwt. 2 qrs. 17 lbs. $\times 6\frac{3}{4}$ of $4\frac{1}{7}$ of $\frac{1}{137}$.

REDUCTION OF FRACTIONS.

No special rule for reducing fractions from one denomination to another is now really required, (though for the sake of convenience rules will be given,) for, reduction of denominate fractions is done by precisely the same rules, as reduction of denominate whole numbers. In fractions, just as in whole numbers, reduction *descending* is done by *multiplying the fraction*, and reduction *ascending* is done by *dividing the fraction*, and the learner is of course supposed now to know how to perform any operation in Multiplication or Division.

To reduce a denominate fraction from one denomination to another.

RULE.—*If the reduction be from a lower to a higher denomination multiply the denominator, (i. e., divide the fraction,) but if from a higher to a lower denomination multiply the numerator, (i. e., multiply the fraction.)*

EXAMPLE 1.—Reduce $\frac{6}{10}$ of an hour to the fraction of a week.

OPERATION.

$$\tfrac{6}{10}\,h = \frac{6}{10 \times 24}\text{ d.} = \frac{1}{10 \times 4 \times 7}\text{ wk.} = \frac{1}{280}.\quad \textit{Ans.}$$

EXAMPLE 2.—Reduce $\frac{7}{5}$ of an acre to the fraction of a yard.

OPERATION.

$$\tfrac{7}{5}\text{ of an acre} = \frac{7 \times 4 \times 40 \times 30\frac{1}{4}}{5}\text{ of a yard.}$$

$$= \frac{33880}{5}\text{ of a yard} = 6776\text{ yards. } \textit{Ans.}$$

EXERCISE 49.

1. Reduce $\frac{3}{11}$ of a day to the fraction of a week.
2. Reduce $\frac{5}{28}$ of a cwt. to the fraction of a quarter.
3. Reduce $\frac{3}{5}$ of $\frac{4}{5}$ of $1\frac{5}{8}$ of a yard to the fraction of an ell Flemish.
4. Reduce $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{15}{33}$ of a mile to the fraction of a yard.
5. Reduce $\frac{4}{5}$ of $\frac{7}{8}$ of $3\frac{1}{3}$ inches to the fraction of a mile.
6. Reduce $\frac{6\frac{1}{2}}{4\frac{3}{4}}$ of 6 oz. to the fraction of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of a scruple.
7. Redude $\frac{4}{15}$ of $\frac{1}{3}$ of $\frac{2\frac{1}{2}}{\frac{1}{4}}$ of a pint to the fraction of $\frac{3}{5}$ of $\frac{4\frac{1}{2}}{7\frac{1}{2}}$ of a peck.

To reduce one denominate number to the fraction of another.

RULE.—*Reduce both quantities to the lowest denomination contained in either.*

Then place that quantity which is to be the fraction of the other as numerator, the other quantity being the denominator.

EXAMPLE 1.—Reduce 3 lbs. 5 oz. to the fraction of 3 lbs. 6 oz. 10 dwt.

OPERATION.

3 lbs. 5 oz. $=$ 820 dwt.
3 lbs. 6 oz. 10 dwt. $=$ 850 dwt.

Therefore 3 lbs. 5 oz. is $\frac{820}{850}$ of 3 lbs. 6 oz. 10 dwt.

EXERCISE 50.

1. What fraction is 10 hours 7 minutes of 1 week 7 hours?

2. What fraction is 9 lbs. 7 oz. 2 grs. of 1 lb. 6 oz. 9 dwt?
3. What fraction is 10 per. 6 yds. 2 ft. 1 in. of 6 roods 4 perches?
4. What fraction is 2 qrs. 1 na. 11 in. of 2 Eng. e. 2. qr. 2 na.
5. Reduce 7 weeks 3 days 4 hours 5 min. to the fraction of a year.
6. Reduce 2 qts. 1 pt. to the fraction of 6 bush. 1 pk.
7. Reduce 11 lbs. 1 oz. to the fraction of 3 qrs. 17 lbs.

EXERCISE 51.

Find the value of the following fractions:

1. $\frac{4}{7}$ of a week.
2. $\frac{1}{6}$ of $\frac{3}{4}$ of a peck.
3. $\frac{3}{8}$ of $\frac{4}{9}$ of $\dfrac{9\frac{1}{2}}{3\frac{1}{4}}$ of a mile.
4. $\frac{3}{11}$ of $8\frac{1}{4}$ hhd.
5. $\frac{3}{5}$ of $\frac{5}{11}$ of $\dfrac{8\frac{1}{2}}{17}$ of a £.
6. $\frac{1}{4}$ of $\frac{4}{11}$ of $6\frac{2}{3}$ of $\frac{4}{17}$ of a cwt.
7. $\frac{3}{8}$ of $\frac{2}{5}$ of an acre.
8. Add together $\frac{4}{9}$ cwt., $8\frac{6}{9}$ lbs., $3\frac{9}{10}$ oz.
9. $3\frac{1}{2}$ Eng. ells + $4\frac{1}{2}$ yds. + $\frac{5}{9}$ nls.
10. $\frac{1}{3}$ wk. + $\frac{1}{4}$ day + $\frac{1}{3}$ hour.
11. $\frac{3}{4}$ of 5s. 6d. + $\frac{3}{8}$ of $\frac{1}{2}$ of 17s. $2\frac{1}{4}$d. + $\frac{5}{9}$ of £5 6s. $7\frac{1}{2}$d.
12. From $8\frac{7}{10}$ cwt. take 2 qrs. $3\frac{4}{9}$ lbs.
13. From 7 weeks take $9\frac{7}{16}$ days.
14. From $7\frac{3}{8}$ oz. take $8\frac{7}{9}$ dwts.
15. $\frac{3}{4}$ of 21s. + $\frac{3}{8}$ of 5s. + $\frac{1}{2}$ of 7s. 6d. − $\frac{3}{4}$ of 2d.

PROBLEMS IN FRACTIONS

BY ANALYSIS.

Analysis is the method of solving questions on general principles; or, the method of determining the operation to be performed simply from the nature of the case.

These *general principles* have already been explained and applied, (pages 70, 71, and ex. 34).

The questions in the following exercise should be thoroughly analysed by the class in the hearing of the teacher, before doing them. This oral analysis will not be dispensed with on any pretext, if due prominence be given to mental training.

Each pupil might be called upon in turn, to analyse the question which the class is about to do; or, the whole class might be called to the black board at once, and a certain time allowed for doing the questions given out; after which, the class being called to their seats facing the board, each pupil should be called to the board in turn to go through the explanation of his question and his work, the members of the class listening, and the teacher observing the manner, the language, the logic, as well as the mechanical operation on the board.

The class should go over the questions a second and even a third time, before leaving them; one day explaining and analysing without performing the operations; another day working out the questions as quickly as possible, without any explanation; the pupil who does the greatest number correctly in a given time, marking first, the next greatest, second, and so on.

EXAMPLE 1.—If $\frac{11}{13}$ of a bushel of wheat cost $\frac{10}{16}$ of a dollar, what will $\frac{24}{48}$ of a bushel cost.

Statement by the pupil.—"Here I have the price of $\frac{11}{13}$ of a bushel, given, to find the price of $\frac{24}{48}$ of a bushel."

I.—ANALYSIS BY THE PUPIL.—"If $\frac{11}{13}$ of a bushel cost $\frac{10}{16}$ of a dollar, then $\frac{1}{13}$ of a bushel would cost the $\frac{1}{11}$ of $\frac{10}{16}$ $=\$\frac{10}{176}$, and $\frac{13}{13}$ or 1 bushel, would cost 13 times as much as $\frac{1}{13}$, *i. e.*, $\$\frac{10}{176}\times 13=\$\frac{130}{176}$. Now, I have the price of

1 bushel to find the price of $\frac{34}{48}$ of a bushel. I will first get the price of $\frac{1}{48}$ of a bushel by dividing the price of one bushel by 48, *i. e.*, $\$\frac{130}{165} \div 48 = \$\frac{130}{7920}$. Now I have the price of $\frac{1}{48}$ to find the price of $\frac{34}{48}$. To do this I will multiply the price of $\frac{1}{48}$ by 34, *i. e.*, $\$\frac{130}{7920} \times 34 = \$\frac{4420}{7920} = \$\frac{221}{396}$, the price of $\frac{34}{48}$ of a bushel, the thing required."

Or, more briefly,

II. Analysis by the Pupil.—"To do this, I will first get the price of one bushel, by dividing the price of $\frac{11}{15}$ of a bushel by $\frac{11}{15}$. Now, I have the price of one bushel, to find the price of $\frac{34}{48}$ of a bushel; to this, I will multiply the price of one bushel by $\frac{34}{48}$, and that will give the price of $\frac{34}{48}$ of a bushel."

Operation. $$\frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{15}}} \times \frac{13}{11} \times \frac{\overset{17}{\cancel{34}}}{\underset{\underset{12}{\cancel{24}}}{\cancel{48}}} = \$\frac{221}{396}$$

Example 2.—A post, standing in a stream, has $\frac{1}{5}$ of its length in the earth, $\frac{2}{3}$ in the water, and 5 feet above the water; what is the length of the post?

Statement by the Pupil.—"Here I have the part of the post which is in the earth, and the part which is in the water, and the number of feet above the water, given, to find the length of the post."

Analysis.—"I will first find what part of the post is in both the land and the water by adding the part in each together, *i. e.*, $\frac{1}{5}$ and $\frac{2}{3} = \frac{13}{15}$. Then the 5 feet above the water must be the remaining $\frac{2}{15}$ of the post. Then as 5 feet is $\frac{2}{15}$, $2\frac{1}{2}$ feet must be $\frac{1}{15}$ and $2\frac{1}{2} \times 15$, *i. e.*, $37\frac{1}{2}$ feet must be the entire length.

Operation.—$\frac{1}{5} + \frac{2}{3} = \frac{13}{15}$, and $\frac{15}{15} - \frac{13}{15} = \frac{2}{15}$, $\frac{5 \times 15}{2} = \frac{75}{2} = 37\frac{1}{2}$ feet, the entire length.

Example 3.—A man can do a certain piece of work in 3 days, and a boy can do it in 5 days, in what time can both together do the work?

ANALYSIS.—The man can do $\frac{1}{3}$ of it in one day, and the boy $\frac{1}{5}$, hence both together could do $\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$ in one day, *i. e.*, $\frac{1}{15}$ in $\frac{1}{8}$ of a day, and therefore the whole work, or $\frac{15}{15}$, in 15 times the $\frac{1}{8}$ of a day, *i. e.* $\frac{15}{8}$ days $=1\frac{7}{8}$ days.

EXAMPLE 4.—Allowing a person to perform a certain journey in $13\frac{1}{2}$ days, by travelling 10 hours a day, in what time ought he to perform the journey by travelling $11\frac{1}{4}$ hours a day?

ANALYSIS.—By travelling 1 hour a day, it would take 10 times as many days as it would by travelling 10 hours a day, *i. e.*, $13\frac{1}{2}\times 10=135$ days. But if he travel $11\frac{1}{4}$ hours a day, it will take him $11\frac{1}{4}$ part as long as when he travelled 1 hour a day, *i. e.*, $135\div\frac{45}{4}=\frac{540}{45}=12$ days.

1. If a man travels $42\frac{2}{3}$ miles in $12\frac{3}{4}$ hours, how many miles does he go per hour?

2. At $17\frac{1}{5}$ dollars per acre, how many acres of land can be bought for $523\frac{1}{3}$ dollars?

3. At $7\frac{1}{6}$ dollars per ton, how many tons of coal can be bought for $52\frac{2}{3}$ dollars?

4. How many yards of linen, $\frac{1}{2}$ yd. wide, will be equal to 30 yards of another kind, which is $\frac{3}{4}$ yd. wide?

5. If iron cost $38\frac{2}{3}$ dollars per ton, how much can be bought for $952\frac{1}{4}$ dollars?

6. One rod contains $16\frac{1}{2}$ feet, how many rods are there in $158\frac{1}{3}$ feet?

7. What number is that, to which if you add $\frac{3}{5}$ of $\frac{5}{6}$ of itself the whole will be 20?

8. What number is that, of which 9 is the $\frac{3}{4}$ part?

9. At $\frac{5}{8}$ of a dollar per yard, what costs $\frac{3}{4}$ of a yard of cloth?

10. A could mow a meadow in 7 days, B in 9 days, and C in 11 days. In what time could the three together mow it?

11. A person bought 19 barrels of apples at $2\frac{1}{4}$ dollars per barrel. Having sold $12\frac{1}{2}$ barrels of them at $2\frac{1}{3}$ dol-

lars a barrel, at what price per barrel must he sell the remainder, to make a profit of $5\frac{5}{8}$ dollars on the whole?

12. Bought at one time 320 acres of land, at $25\frac{1}{4}$ dollars an acre; and at another time 275 acres, at $31\frac{1}{4}$ dollars an acre. If $\frac{3}{5}$ of the whole quantity were sold at 20 dollars, and the remainder at 30 dollars an acre, what would be the gain or loss?

13. If three men can plough $15\frac{1}{2}$ acres of ground in 4 days, how much ought one man to plough in 1 day? How much ought five men to plough in $7\frac{3}{8}$ days?

14. A purchased of B 40 yards of cloth for 260 dollars. He then sold to C $\frac{3}{5}$ of his purchase at a profit of $\frac{1}{2}$ dollar per yard, and the remainder to D at a loss of $\frac{1}{3}$ dollar per yard. What did A gain or lose by these several transactions?

15. If you pay $\frac{1}{4}$ of $\frac{7}{11}$ of $4\frac{2}{3}$ cents a piece for pears, how many can you buy for $\frac{2}{3}$ of $\frac{5}{8}$ of $18\frac{3}{4}$ cents?

16. A, B, and C together can do a piece of work in 5 days; A and B together can do it in 9 days. In what time could C alone do it?

17. What number must be taken from $256\frac{12}{35}$, that the remainder may be $116\frac{3}{8}$?

18. If $\frac{3}{4}$ of $\frac{5}{8}$ of an acre of land sells for \$18 $18\frac{3}{4}$ cents, what would a lot containing 7 a. 2 r. 13 p. bring at that rate?

19. Bought a piece of silk containing $96\frac{3}{4}$ yards, and having used $\frac{3}{5}$ of it, sold $\frac{3}{4}$ of the remainder at $1\frac{1}{4}$ dollar a yard, the remnant was put at $\frac{7}{8}$ dollar a yard. How much did the parts sold come to?

20. If the sum of $87\frac{11}{12}$ and $117\frac{13}{49}$ is divided by their difference, what will be the quotient?

21. If the difference between $91\frac{1}{4}$ and $95\frac{3}{8}$ is multiplied by $\frac{5}{8}$ of the former, what will be the product?

22. A man owned $\frac{7}{15}$ of a ship's cargo; but in a gale the captain was obliged to throw overboard goods to

the amount of $\frac{4}{9}$ of the whole cargo. What part of the loss must this man sustain?

23. Find the value of $\dfrac{5\frac{4}{5}-2\frac{1}{3}}{3\frac{3}{4}+\frac{9}{20}}$ of $\dfrac{4\frac{1}{2}+5\frac{12}{25}}{4\frac{1}{20}}$ of $\dfrac{2\frac{2}{3}+1\frac{2}{3}}{7\frac{13}{24}-2\frac{1}{4}}$.

24. If $\frac{9}{16}$ of a ton is worth £4 10s., what is the value of $\frac{1}{3}$ of it?

25. After taking out of a purse $\frac{2}{3}$ of its contents, $\frac{4}{5}$ of the remainder was found to be 13s. $5\frac{1}{2}$d.; what sum did it contain at first?

26. The dimensions of a room are $29\frac{1}{2}$ ft. by $11\frac{1}{4}$ ft.; what length of carpet, $\frac{5}{8}$ yd. wide, will cover it? and what will be the expense of it, at $3\frac{3}{4}$s. per yard?

27. A ship is worth £16000, and a person, possessed of $\frac{5}{16}$ of it, sells $\frac{2}{3}$ of his share; what share has he remaining, and what is it worth?

28. Express 4 bus. 1 pk. 1. gal. 2 qts. as a fr. of a qr.; and reduce 5 cwt. to lbs. Troy.

29. If $\frac{1}{3}$ of a ship be worth £36 10s. $7\frac{1}{2}$d., what share will cost £125 5s.?

30. Multiply $3\frac{3}{20}$ by $15\frac{5}{7}$, and divide $\dfrac{2}{3\frac{3}{4}}$ by $\dfrac{2\frac{3}{4}}{3}$; and add together the sum and difference of these results.

DECIMALS.

The pupil is supposed, from having mastered the explanations and rules, and done the exercises already introduced, to be able now to read and write, add, subtract, multiply, and divide decimals.

A few additional exercises on these rules are here introduced in connection with the principles on which the rules are founded, before completing the subject of decimals.

I.—*Remember, that, in reading and writing decimals, there are two things to be attended to.*

1. The *number*, (which is read and written, just as in whole numbers, altogether independently of the decimal point.)
2. The *name*, (which is determined by the number of figures in the decimal, on the principle that one decimal figure has *two* (*i. e.* a unit and one cypher) for a denominator, *two* has three, three has four; or, determined by the name of the figure farthest to the right from the decimal point.)

EXAMPLES.—·15, No. is 15, name, is *hundredths*; read *fifteen hundredths.*

·179, No. is 179, name, is *thousandths;* read *one hundred and seventy-nine thousandths, &c.*

OBSERVE,—Whole numbers and decimals may be read either separately or together, giving the whole numbers the name of the lowest decimal; thus, 6·2 is read 6 and 2 tenths, or 62 tenths,—764·15 is read 764 and 15 hundredths, or seventy-six thousand four hundred and fifteen hundredths.

EXERCISE 52.

Read and write the following decimals in as many different ways as possible.

1. ·7864, 78·6237, and 8064·2987432.
2. Seventy-nine millionths—seventy-nine million billionths.
3. Two thousand ten billionths—two billion ten thousandths.

EXERCISE 53.

II.—Remember, that, *the greater the multiplicand and the multiplier, the greater the product; and the less the multiplicand and multiplier, the less the product.* This being the case—

1. Compare the products of 2 × 4, ·2 × 4, and ·2 × ·4, and show, according to this principle, why these

three products differ as they do, especially with reference to the number of decimals to be pointed off.

2. Do the same with 3×6, $\cdot 03 \times 6$, and $\cdot 03 \times \cdot 06$.
3. Repeat the rule for multiplication of decimals, and prove its correctness by this principle, (see note bottom pages 27 and 28.)

Find the products of

4. $\cdot 002 \times \cdot 003 \times 64\cdot 23 \times \cdot 0007 \times 760\cdot 3$.
5. $\cdot 5 \times \cdot 006 \times 2\cdot 7 \times 0053$ and $\cdot 03 \times \cdot 4 \times \cdot 0067$.

III.—Remember, that, *the less the divisor and the greater the dividend, the greater the quotient, while, the greater the divisor and the less the dividend, the less the quotient.* This being the case—

EXERCISE 54.

1. Compare the quotients of $8 \div 2$, $\cdot 8 \div 2$, $8 \div \cdot 2$, $\cdot 8 \div \cdot 2$, and show, according to this principle, why these four quotients differ or agree as they do, especially with reference to the number of decimals to be pointed off.
2. Do the same with $4 \div 2$, $\cdot 4 + 2$, $4 \div \cdot 2$, and $\cdot 4 \div \cdot 2$.
3. Repeat the rule for division of decimals, and prove its correctness by this principle, (see page 37.)
4. Find the quotient of $\cdot 006 \div 27$, $6 \div \cdot 27$, $\cdot 0052 \div 27$.
5. $\cdot 13 \div 7\cdot 864$, $7\cdot 862 \div 786\cdot 2$, and $\cdot 0020 \div \cdot 02$.

IV.—Remember, that, *multiplying the dividend and then dividing the quotient by the same number, leaves the quotient the same in value as if neither had been done.*

EXAMPLE. $\frac{8}{4} = 2$ and $\frac{8 \times 3}{4} \div 3 = 2$.

EXERCISE 55.

1. Show how this principle applies to prove the following:—

RULE, *for reducing a vulgar fraction to a decimal. Annex ciphers to the numerator, then divide by the denominator, and point off as in division.*

EXAMPLE.—Reduce, $\frac{3}{4}$ to its decimal.

$$\frac{3}{4} = \frac{3{\cdot}00}{4} = {\cdot}75.$$

Reduce to their equivalent decimals:

2. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{7}{8}$, $\frac{13}{25}$.
3. $\frac{187}{240}$, $\frac{216}{25}$, $\frac{31423}{71419}$.
4. $6\frac{473}{[illegible]}$, $17\frac{10}{867}$, $25\frac{13}{5000}$.
5. $8\frac{1}{3}$, $12\frac{2}{7}$, $15\frac{2}{3}$.
6. $27\frac{1}{13}$, $96\frac{15}{16}$, $112\frac{17}{33}$.

In doing this reduction, sometimes the same figure or figures will constantly recur in the quotient, and the division would never terminate.

EXAMPLE, $\frac{1}{3} = {\cdot}333$, &c. $\frac{4}{7} = {\cdot}\dot{5}7142\dot{8}$, &c.

When one figure repeats, as in the first example, the decimal is called a *single repetend*, and it is expressed by writing a dot over the figure that repeats, thus, ${\cdot}\dot{3}$. When more than one figure repeats, the decimal is called a *circulating decimal* or *compound repetend*, and it is expressed by writing a dot over the first and last figures of the *period or circle* (as the set of figures that repeats is called) thus, ${\cdot}\dot{5}7142\dot{8}$.

Sometimes, the first figure after the decimal point is the figure, or, the first of a set of figures that repeats; this is called a *pure* repetend if only one repeats, and a *pure circulating decimal* if more than one repeats. If the figures do not commence to repeat immediately after the decimal point, the decimal is called a mixed repetend, or, mixed circulating decimal.

EXAMPLE.—${\cdot}\dot{3}$ is a pure repetend; ${\cdot}\dot{1}5\dot{6}$ is a pure circulating decimal; ${\cdot}57\dot{2}$ is a a mixed repetend; ${\cdot}01\dot{2}3\dot{6}$ is a mixed circulating decimal.

To reduce a *pure repetend, or a pure circulating decimal* to a vulgar fraction:—

RULE.—*Take the decimal itself for the numerator, and as many 9's as there are figures in the decimal for a denominator.*

EXAMPLE.—$\cdot\dot{8} = \frac{8}{9}$, $\cdot\dot{1}5\dot{6} = \frac{156}{999}$.

To reduce a mixed repetend to a vulgar fraction.

RULE.—*Subtract the figures that do not repeat from the whole decimal, to get the numerator; and for the denominator, write as many 9's as there are repeating figures, followed by as many 0's as there are figures that do not repeat.*

Reduce $\cdot3\dot{4}\dot{5}$ and $\cdot12\dot{3}\dot{4}$ to vulgar fractions.

EXAMPLE.—$\cdot3\dot{4}\dot{5} = \frac{342}{990}$, $\cdot12\dot{3}\dot{4} = \frac{1222}{9900}$.

EXERCISE 56.

Reduce to equivalent vulgar fractions.

1. $\cdot\dot{4}\dot{3}$, $\cdot\dot{1}5\dot{3}$. 2. $\cdot\dot{0}3\dot{7}$. $\cdot\dot{1}3\dot{8}$.
3. $\cdot\dot{4}2\dot{5}$, $\cdot1\dot{6}$. 4. $\dot{1}67\dot{8}$, $7\cdot16\dot{1}5\dot{1}$.

To add, subtract, multiply, or divide pure or mixed repetends, or circulating decimals,

RULE.—*First reduce them to their equivalent vulgar fractions, then proceed as with any other vulgar fractions.*

EXERCISE 57.

1. $27\cdot2\dot{3} + \cdot2\dot{6} + 7\cdot7\dot{2} + \cdot29\dot{7} + 3\cdot\dot{9}7\dot{3} + 4\cdot7\dot{5}$.
2. $\cdot\dot{3} - \cdot\dot{0}\dot{9}$, $\cdot\dot{0}\dot{4} - \cdot00\dot{7}6923\dot{8}$.
3. $37\cdot2\dot{3} \times \cdot2\dot{6}$, $7\cdot\dot{7}\dot{2} \times 4\cdot7\dot{5}$.
4. $\cdot\dot{6}1\dot{4} \div 2\cdot7\dot{6}\dot{6}$, $6\cdot7\dot{1} \div \cdot03\dot{6}$.

To find the value of a given decimal in integers of the lower denominations—

RULE.—*Multiply the given decimal by as many of the lower denomination as it takes to make one of the same, pointing off from the product as in multiplication; then proceed as before with the decimal part, and so on till the lowest denomination is reached.*

EXAMPLE.—Find the value of 23·43 of a day.

```
Days     23·43 and decimal of days.
            24
         -----
           172
           86
         -----
Hours    10·32 and decimal of hours.
            60
         -----
Minutes  19·20 and decimal of minutes.
            60
         -----
Seconds  12·00
```

Hence, 23·43 days = 23 d. 10 h. 19 m. 12 s.

EXERCISE 58.

Find the value of

1. ·625 cwt.
2. $\cdot 72\dot{6}$ gals.
3. $\cdot 33\dot{7}\dot{5}$ tons.
4. $\cdot\dot{0}5$ acres.
5. 1·85 of 3s. 4d.
6. $\cdot 131\dot{2}\dot{5}$ of \$5.
7. ·325 of $1\frac{1}{2}$ ton.
8. $\cdot\dot{1}7\dot{6}$ of 1 fur. 36 p. 2 yds. 5 in.

To reduce a denominate number to the decimal of another given denominate number,

RULE.—*Beginning with the lowest denomination, reduce it to the decimal of the next higher, writing it after the higher, to which it was reduced; then, reduce this as before and annex it to the next higher, and so on, ascending to the required denomination.*

EXAMPLE I.

Reduce £4 6s. 9d. to the decimal of a £.

OPERATION.

12)9 d.

20)6.75 s.

4.3375 dec. of a £.

EXAMPLE II.

Reduce 3 pks. 1 gal. 2 qts. 1 pt. to the decimal of a bushel.

OPERATION.

2)1 pt.

4)2.5 qts.

2)1.625 gal.

4)3.8125 pks.

.953125 dec. of a bush.

EXERCISE 59.

1. Reduce 9 oz. 2 drs. to the decimal of a lb.
2. " 3 fur. 33 yds. to the decimal of a mile.
3. " 1 cwt. 3 qrs. 7 lbs. to the decimal of $2\frac{1}{2}$ tons.
4. " $4\frac{1}{2}$ lbs. to the decimal of 3 qrs. 12 lbs.
5. " 3 hrs. 3 min. 2 sec. to the decimal of a day.
6. " $3\frac{1}{2}$ inches to the decimal of $\frac{1}{4}$ mile.
7. " 22 guineas to the decimal of £25.
8. " 17 yds. 1 ft. 6 in. to the dec. of a mile.
9. Paid 36c. for .625 yard of cloth; what was that a yard?
10. Paid $\frac{5}{8}$ dollar for .125 bbl. of flour; how much was that per barrel?
11. If you walk .965625 mile per hour, how far can you walk in a week?
12. What cost .778125 ton of butter, at 2s. per pound?
13. .815625 lb. of silver was sold at 5s. per grain; what did it come to?
14. Bought .45683 acre of land, at .5 dollar a foot; what did it come to?

SECTION IV.

RATIO, PROPORTION, PARTNERSHIP.

RATIO.

The Ratio of one number to another is the relation expressed by the quotient obtained by dividing the former by the latter.

Thus, the *Ratio* of 6 to 12 $= \frac{1}{2}$, but the ratio of 12 to 6 $= 2$.

The former term of the ratio is called the *antecedent*, the latter is called the *consequent.* The ratio between two numbers is usually expressed thus, 6 : 12, which signifies the same as $6 \div 12$, or $\frac{6}{12}$.

When the consequent is *equal* to the antecedent, the ratio is called a ratio of *equality*, as 6 : 6; when the consequent is *greater* than the antecedent, it is called a ratio of *greater inequality* or a *rising* ratio, as 6 : 12; when the consequent is *less* than the antecedent, the ratio is called a ratio of *less inequality*, or a falling ratio, as 12 : 6.

Ratios are compounded by multiplying all the antecedents together for a new antecedent, and all the consequents together for a new consequent.

The Ratios 2 : 4, 6 : 8, 3 : 2, when compounded = 36 : 64, which is called a *compound ratio.*

The *value* of a ratio is found by dividing the antecedent by the consequent.

EXERCISE 60.

1. What is the value of each of the ratios, 7 : $6\frac{1}{4}$, 2 : 15, $13\frac{1}{4}$: 6.

2. Which of the following ratios is the greatest and which is the least, $7\frac{3}{8} : 5$, $10\frac{1}{2} : 7$, $12 : 9\frac{3}{7}$.

Compound together the following ratios:

3. $2 : 3$, $8 : 7$, $1\frac{1}{2} : 6$.
4. 2 lb. 8 oz. : 1 dwt. 3 qrs., 2 tons 1 cwt. : 2 qrs. 16 lbs.
5. 2 years 4 months : 3 weeks 4 days, 3 months 4 days : 13 hours 27 minutes.

PROPORTION.

A Proportion consists in the combination or union of two *equal* ratios by the sign (: :) indicating their equality, thus, 6 : 18 : : 8 : 24, read, 6 is to 18 as 8 is to 24, meaning that 6 divided by 18, gives the same quotient as 8 divided by 24, or, $\frac{6}{18} = \frac{8}{24}$.

In solving a problem in proportion, there are *two things* to be attended to, viz.,—1st. The arrangement of the terms, or proper statement of the question; and 2nd. The working out of the question, or the operation by which the answer is obtained.

First, The proper statement of the question.

Example.—If 7 men can build a house in 28 days, in what time can 17 men build it?

Observe.—In this example, and in every question in simple proportion, there are three terms given to find a fourth. Two of the three are alike, viz.: (in this example,) the 7 men and the 17 men, forming a complete or perfect ratio; the other is the given term of the incomplete ratio, the *mate*, as it were, of the fourth, or required term. This odd, or unmated term, the term which is of the same name as the thing required, or answer, being the term of the *incomplete ratio*, must therefore always be the third term. Hence, *of the three given numbers, always make that the [illegible] term, which is of the same name as the answer.* Then, in order to arrange the other two terms correctly, *considering whether the answer is to be greater or less than the third term; if greater, it is a rising ratio, if less, it is a falling ratio.*

Applying these remarks to the question before us, the learner will observe, not only the *statement*, but how every question should at first be reasoned out before doing it on the slate.

The pupil after reading the question distinctly in the hearing of the class, as follows:—"If 7 men can build a house in 28 days, in what time can 17 men build it?" proceeds,—"Here I have the number of days," (or whatever may be required in the question,) "in which 7 men can do a piece of work, given, to find the *number of days* in which 17 men can do the same work. As days are required, I will put days in the third term; and as I know that it will take less days for 17 men to do a piece of work than for 7 men to do the same work, therefore it is a falling ratio."

The question stands when stated,

Men. Men. Days.
17 : 7 : : 28 : *Ans.* Read, 17 men is to 7 men, as 28 days is to the answer.

In the same manner reason out, and state the following questions.

EXERCISE 61.

1. If I buy 7 lbs. of sugar for 75 cents, how many pounds can I buy for $6.

2. If a family of 10 persons use 8 bushels of wheat in a month, how many bushels will serve them when there are 30 in the family?

3. If a person, whose rent is $145, pays $12·63 taxes, how much should a person pay whose rent is $378?

4. If I give $6 for the use of $100 for 12 months, what must I give for the use of $357.82 the same time.

* Observe, although in this, and in some of the other questions, the whole three terms have the same general name, still, it is only the two terms of the complete ratio that are precisely alike. The term of the incomplete ratio is essentially different from the other two. In this question, it is *taxes*, while the other two are *rent*. In the 5th question, again, although the three terms are all *length*, yet only one is *real length*, or length of substance, the other two are length of *shadow*.

5. If a staff 5 ft. 8 in. in length, cast a shadow of 6 feet, how high is that steeple whose shadow measures 153 feet?

Now, with reference to the second thing to be attended to, viz., the working out of the question after it is stated,

OBSERVE.—The first thing is to see whether the *first* and *second* terms are in the same denomination; if not, they must be reduced to the same name, for there could be no ratio between them, if, for instance, one was *ounces* and the other *pounds*, or one *years* and the other *months*. If the *third term* has several denominations, as cwts., qrs., lbs., it may or may not be reduced to the lowest denomination, this being merely a matter of convenience.

In order to understand why we must multiply the second and third terms together, and divide the product by the first term to get the answer, just consider that,

In every proportion, *the product of the means* (or middle terms, that is the second and third,) *is equal to the product of the extremes*, (that is the first and fourth terms); for instance, in the proportion, $6:9::18:27$, $6 \times 27 = 9 \times 18 = 162$. This must necessarily be the case; for, the two ratios are equal—that is, the two questions in division have equal quotients, and the product of the *means* is the product of the divisor of the one into the dividend of the other, while the product of the *extremes* is the product of the other divisor and dividend. For example, $6:8::12:16$—that is $\frac{6}{8} = \frac{12}{16}$, *therefore* $6 \times 16 = 12 \times 8$, reducing the fractions to a common denominator $\frac{12}{16} = \frac{12}{16}$, the means and the extremes are now seen to be actually the same when a common unit of comparison is adopted, the means, being 12×16, the extremes also being 12×16. Hence:

When one of the two ratios of a proportion, and one term of the other ratio are given to find the other, or the unknown term, (which is the case in every problem solved by proportion,) this law of proportion—viz., "*The product of the means is equal to the product of the extremes*", enables us to find the required term of the incomplete ratio. For, since we have one of the extremes, viz., the

first term, and since, "the product of the means is equal to the product of the extremes," hence, when we get the product of the means we have a product and one of the factors, viz., the first term, given to find the other, viz., the fourth term, of course, we have only to divide the product by the *known* factor or first term to find the *unknown* factor or fourth term. Hence, to do any question after it is stated and the first and second terms reduced to the same name, *multiply the second and third terms together, and divide the product by the first term,*—the quotient will be the fourth term, that is, the required term of the incomplete ratio, or the answer.

EXAMPLE.—If I buy 7 lbs. of sugar for 75 cents, how many can I buy for $6.

OPERATION.

75 c.	: $6	: :	7 lbs.
	100		600
	———		———lbs.
	600		75)4200(56
			375
			———
			450
			450

Here, after reducing the second term to the same name as the first, viz., to cents; *as the product of the means is equal to the product of the extremes,* we multiply 600 and 7, (*the means*) together to get the product of *the extremes*. Then we have the product of the extremes, viz., 4200, and one of the extremes, viz., 75, (the first term) given to find the other, of course we divide the product 4200 by the known factor, 75, and get the other, the unknown factor, or fourth term, 56 lbs. the Ans.

What has been said with reference to the statement and working out of questions in proportion, may now be expressed in the form of a rule.

RULE FOR PROPORTION.—*Of the three given numbers, make that one the third term which is of the same name as the one required in the answer.*

Then consider from the nature of the question whether the answer will be greater or less than this term. If greater, arrange the two remaining terms as a rising ratio; if less, arrange them as a falling ratio.

Then reduce the first and second terms to the same name, and, either leave the third term as it is, or reduce it to its lowest denomination, as may be thought most convenient.

Then multiply the second and third terms together, and divide the product by the first term. The quotient will be the answer, in the same name as the third term was reduced to, and may be reduced to any denomination required.

EXERCISE 62.

1. If 6 men can reap 20 acres in 1 day; how many men can reap 45 acres in the same time?
2. If 3 yards of cloth cost \$2.62; how many yards can be bought for \$7?
3. In how many days can a man travel 75 miles, at the rate of 7300 miles per year?
4. If \$114 be paid for 52 cwt. 1 qr. 4 lbs. of flour; what would 122 cwt. cost?
5. If 57 cwt. of sugar cost \$216; what would 95 tons 3 cwt. 2 qrs. 17 lbs. cost?
6. If 275 quires of paper cost \$33.15; what would 990 reams cost?
7. If 96 men reap 40 acres of grain in a week; how many men would reap 65 acres, 3 rods, 16 per. 20 yds. in the same time?
8. There are two numbers in the ratio of 5 to 3. the larger is 85; what is the smaller?
9. There was a certain building raised in 8 months by 120 workmen; but the same being burned, it is required to be built in 2 months 17 days. How many men must be employed about it?
10. There is a cistern having a pipe which will empty it in 10 hours; how many pipes of the same capacity will empty it in 24 minutes?
11. A garrison of 1200 men has provisions for 9 months, at the rate of 14 oz. per day; how long will the provisions last, at the same allowance, if the garrison be reinforced by 400 men?
12. If a piece of land, 40 rods in length and 4 in breadth, make an acre; how wide must it be when it is but 25 rods long?

13. If a man perform a journey in 15 days, when the days are 12 hours long; in how many will he do it when the days are but 10 hours long?

14. If a field will feed 6 cows 91 days; how long will it feed 21 cows?

15. If I can walk from Kingston to Toronto, a distance of 180 miles, in 46 hrs. 18 min.; in what time can I walk from Toronto to Montreal, the distance being 333 miles?

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16. A farmer sold $\frac{1}{3}$ of his land to A, $\frac{1}{4}$ of it to B, and the remainder, which was 100 acres, to C; how much land did the farmer own?

17. A cistern whose capacity is 3000 gals., is supplied with water by a pipe which pours into it 7 gal. per minute. By leakage the cistern will lose, during the time of filling, at the rate of $2\frac{1}{2}$ gals. per minute; in what time will the cistern be filled.

18. A post standing in a stream, has $\frac{1}{5}$ of its length in the earth, $\frac{2}{3}$ in the water, and 5 feet above the water; what is the length of the post?

19. A person failing in business owes $5000, and is able to pay only $2000; how much can he pay per dollar to his creditors, and how much should that creditor receive to whom he owes $1000?

20. How many yards of linen $\frac{1}{2}$ yd. wide, will be equivalent to 30 yds. of another kind which is $\frac{3}{4}$ yd. wide?

21. A traveller having gone 375.5 miles on his journey, finds that $\frac{3}{8}$ of it remains to be travelled; what was the length of his journey?

22. A and B depart from the same place, and journey in the same direction; A starts 3 days before B, and goes 30 miles per day; B follows at the rate of $33\frac{1}{3}$ miles per day; in how many days will the latter overtake the former?

NOTE.—The first 15 questions of this Exercise may be done without a knowledge of Vulgar and Decimal Fractions. The remaining questions are designed for exercises in Fractions as well as Proportion.

23. Allowing a man to do certain work in 3 days, and a boy to do it in 5 days; in what time ought both together to do the work?

24. A can dig a ditch in 5 days, B in 6 days, and C in 8 days; in what time could the three together dig the ditch?

25. Two masons together built a wall in 10 days; one of them could have built the wall himself in 15 days; in how many days could the other have done it?

26. If 1 oz. of gold is worth \$14.89; what is the value $\frac{1}{8}$ of $\frac{5\frac{1}{2}}{2}$ of $\frac{2}{5\frac{1}{2}}$ of 3 lbs.?

27. If the rent of 20 ac. 3 rd. 2 per. be \$50; what will be the rent of $\frac{5}{8}$ of $\frac{2\frac{1}{2}}{3}$ of 3 ac. 3 r.?

---

28. If 9·35 lbs. of rice cost \$0.62; how many pounds can be bought for \$9.73?

29. If 3 ac. 2 r. of land is worth \$265.35; what is the value of 3·63 acres?

30. If $\cdot\dot{3}$ ozs. of silver is worth \$$0\cdot23\dot{6}$; what is the value of $\frac{4}{5}$ of $\cdot\dot{1}29\dot{8}$ lbs.?

31. If a grocer use instead of a gallon, a measure which contains $\cdot98\dot{7}$ gal., what would be the *true* measure of 100 of these false gallons?

32. What is the value of 3 yds. 2 qrs. 2 nls. of cloth, if ·00001001 yds. cost \$·00030107?

33. A testator bequeathed $\frac{1}{2}$ of his estate to his only son, $\frac{3}{7}$ of the remainder to his only daughter, and the remainder, which was \$5000, to his widow; what was the value of his estate?

34. If $\frac{3}{4}$ of $\frac{4}{5}$ of $\cdot\dot{3}4\dot{5}$ ac. cost \$18·1875; what cost 7 ac. 2 r. 13 per.?

35. If 9 sheep cost £3 13s. 4½d.; what cost $\frac{3}{8}$ of $\frac{1}{25}$ of $\frac{5\frac{1}{2}}{4\frac{1}{2}}$ of a flock containing 1000? (Expressed in decimal currency).

36. If the freight on a ton of merchandise is £1 3s. $2\frac{1}{2}$d.; how many tons can be paid for with $\frac{1}{4}$ of $\frac{1}{6}$ of $\frac{2}{3}$ of \$48.56?

37. If it takes 40 yds. of carpet to cover $\frac{3}{10}$ of the floor of a room when the carpet is 27 inches wide; how many yds. will cover the same, the width being $\frac{3}{8}$ of $\frac{8}{9}$ of 2 ft. 9 in.?

38. What cost 90 tons of hay, at the rate of $\frac{3}{8}$ of a cwt. for \$$9.\dot{3}2\dot{5}$?

39. A can build a well in 5 days, B in 8 days. A began and wrought for 3 days and then left. B commenced to finish it; in what time can he do so? (expressed in the decimal of a day and of a year.)

COMPOUND PROPORTION.

When the relation of the required quantity to the given quantity of the same name depends upon two or more conditions, we have a Compound Proportion; that is, an equality between a Compound and a Simple Ratio.

A question in Compound Proportion is stated just as a question in Simple Proportion; *i. e.*, each simple ratio found in the question is made a rising or falling ratio, without any reference to its connection with the other simple ratios with which it is to be compounded when all are arranged.

EXAMPLE.—If 264 men, in 5 days of 12 hours each, can dig a trench 240 yards long, 3 wide, and 2 deep; in how many days, of 9 hours long, will 24 men dig a trench, 420 yards long, 5 wide, and 3 deep?

In this example, the number of days required depends upon five conditions, viz., the number of men, the length of the days, the length, the width, and the depth of the trench. As days are required, of course the 5 days go in the third term; then, 1st, will 24 men require more or less days than 264? More, of course,—hence, the men form a rising ratio. Then, 2nd, will 9 hours long require more or less days than 12 hours long? More,—hence, the

hours form a rising ratio; and so on with the other three, viz., the length, the width, and depth. The question stands when stated,

$$
\left.\begin{array}{lrcl}
 & & & 11 \\
\text{Men,} & \cancel{24} & : & \cancel{264} \\
 & 3 & & \cancel{4} \\
\text{Hours,} & \cancel{9} & : & \cancel{12} \\
 & \cancel{4} & & 7 \\
\text{Length} & \cancel{240} & : & \cancel{420} \\
\text{Width,} & \cancel{8} & : & 5 \\
\text{Depth,} & 2 & : & \cancel{8}
\end{array}\right\} 5 \text{ days.}
$$

$$
\begin{array}{rcl}
6 & : & 385 \quad :: 5 \\
 & & 5 \\ \hline
 & 6) & 1925 \\ \hline
 & & 320\tfrac{5}{6} \text{ days. } \textit{Ans.}
\end{array}
$$

Before arranging each ratio, the terms must be reduced to the same name if not so already, and before compounding the ratios, all factors common to the first and second terms should be cancelled; then, having compounded the ratios, we proceed just as in simple proportion.

Hence, we have the following rule for Compound Proportion;

RULE.—*Of the given numbers, make that the third term which is of the same name as the answer; then, having reduced each pair of corresponding terms to the same denomination, arrange them as in simple proportion; then, having cancelled as much as possible, multiply together the third term, and all the second terms, and divide the result by the product of the first terms.*

EXERCISE 65.

1. If 5 bushels of wheat serve 9 persons for 22 days, how long will 20 bushels serve 6 persons?
2. If 12 horses in 5 days plough 11 acres, how many horses would plough 33 acres in 18 days?
3. If 8 men earn £9 in 5 days, how much would 32 men earn in 24 days, working at the same rate?
4. If 14 horses eat 56 bushels of oats in 16 days, how many might be kept on 120 bushels for 24 days?

5. If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of $12\frac{1}{2}$ sheets?

6. If 12 horses plough 11 acres in 5 days, how many will plough 33 acres in 18 days?

7. If a person earn 16 guineas in 108 days, how many sovereigns would he earn at the same rate in 270 days, 20 guineas being equal to 21 sovereigns?

8. If a garrison of 1800 men eat 100 barrels of flour in 35 days, how many men would eat 200 barrels in 45 days?

9. If 1000 men, besieged in a town, with provisions for 28 days, at the rate of 18 ounces a day for each man, be reinforced by 600 men; how many ounces a day must each man have that the provisions may last them for 42 days?

---

10. If 7 men can mow 84 acres in 12 days of $8\frac{1}{4}$ hours each; how many can be mowed by 20 men in 11 days of $7\frac{1}{3}$ hours each?

11. If 8 men can dig a trench 100 ft. long, 3 ft. broad, and 4 ft. 6 in. deep in 9 days; how many will be required to dig a trench 80 ft. long, 5 ft. broad, and 2 ft. deep, in $5\frac{1}{3}$ days?

12. If 7 masons can erect a certain piece of wall in $20\frac{5}{8}$ days of $9\frac{3}{5}$ hours each; how long would it take 3 masons to do $2\frac{3}{4}$ of the same work, reckoning 12 hours to the day?

13. If 6 iron bars, 4 ft. long, 3 in. broad, and 2 in. thick, weigh 288 lbs.; how much will 15 weigh, each $6\frac{1}{2}$ ft. long, 4 in. broad, and 3 in. thick?

14. How many pounds of thread will it require to make 60 yards, of 3 quarters wide, if 7 pounds make 14 yards, 6 quarters wide?

15. How much hay will 32 horses eat in 120 days, if 96 horses eat $3\frac{3}{4}$ tons in $7\frac{1}{2}$ weeks?

16. If \$2.45 will pay for painting a surface 21 feet long and $13\frac{1}{3}$ feet wide, what length of surface that is $10\frac{2}{3}$ feet wide, can be painted for \$31.72?

17. If $\frac{3}{4}$ of a lb. of tea cost $\frac{1}{2}$ of a dollar, what would $\frac{5}{8}$ths of a cwt. of sugar cost, tea being 60 times the price of sugar, weight for weight?

18. If 36 oxen eat the grass of 9 acres in 15 weeks, and 24 oxen eat the same in 27 weeks; how many oxen would eat the grass of 20 acres in 30 weeks, supposing the grass to grow uniformly?

19. If 100 men, by working 6 hours in each day, can in 27 days dig 18 cellars, each 40 ft. long, 36 ft. wide, and 12 ft. deep; how many cellars that are each 24 ft. long, 27 ft. wide, and 18 ft. deep, can 240 men dig in 81 days, by working 8 hours a day?

20. If the 8d. loaf weighs 5.3 oz. when wheat is at $13.50 per quarter; what should be the price of wheat when the 6d. loaf weighs 32 oz. 8 dwt.?

21. If 24 pioneers, in $2\frac{1}{2}$ days, of $12\frac{1}{2}$ hours long, can dig a trench 139.75 yds. long, $4\frac{1}{2}$ yds. wide, and $2\frac{1}{4}$ yds. deep; what length of trench will 90 pioneers dig in $4\frac{1}{3}$ days of $9\frac{2}{3}$ hours long, the trench being $4\frac{7}{8}$ yds. wide and $3\frac{1}{3}$ yds. deep?

22. A contractor engaged to pave 15 miles of road in 12 months, and for that purpose employed 100 men. Seven months have now elapsed, and but 6 miles of the road have been completed; how many more men must be employed to finish the work in the time prescribed?

23. If 3 men can cradle 97·534 acres in $3\frac{5}{9}$ days, of $7\frac{2}{3}$ hours each; how many men will cradle the same in $17\frac{7}{9}$ days, of 10·4 hours each?

24. A garrison of 500 men has provisions for 50 days, allowing each man 21·35 oz. per day,—they are reinforced by 1,000 men at the end of 20 days; how much per day must be allowed to each in order that the provisions may last for the 50 days?

25. If 27 men can mow 20 acres of grass in $5\frac{5}{6}$ days, working $3\frac{3}{5}$ hours a day, how many acres can 10 men mow in $4\frac{1}{2}$ days, by working $8\frac{1}{2}$ hours a day?

26. If a footman travel 341 miles in $7\frac{1}{3}$ days, travelling $12\frac{1}{7}$ hours each day, in how many days, travelling $10\frac{1}{3}$ hours a day, will he travel 155 miles?

27. If a person gain $8\frac{1}{3}$ per cent. by selling apples at the rate of 8 for $6\frac{1}{2}$ cents; how much does he gain per cent. by selling them at the rate of 3 for $2\frac{1}{2}$ cents?

28. If 3 men and 4 women can do $\frac{4}{5}$ of a piece of work in 56 days; in how many days will 1 man and 1 woman do $\frac{3}{8}$ of it?

SIMPLE PARTNERSHIP.

SIMPLE PARTNERSHIP is the method of distributing the Profits or Losses of a Firm, or Company, equitably among the partners, when the stocks or sums contributed by the several partners continue in trade for the same time.

The sum of the stocks or shares of all the partners is called the *Stock* or *Capital* of the business.

The profit or loss of the company is distributed to each partner by the following

RULE.—*The whole capital is to each partner's share of the capital, as the whole gain or loss is to each partner's gain or loss.*

EXAMPLE.—A, B, and C enter into partnership with a capital of \$1000, of which A gave \$400, B gave \$350, and C gave \$250. They gain \$540. What is each partner's share of the gain?

By the rule,—

whole capital : A's share : : whole gain : A's gain,

or, in this example, \$1000 : \$400 : : \$540 : A's gain.

$$\text{or,}\quad \frac{\$400 \times 540}{1000} = \$216 = \text{A's gain.}$$

Again, to find B's share, substitute his share of the capital (\$350) for A's in the above proportion.

\$1000 : \$350 : : \$540 : \$189, B's share of gain.

and \$1000 : \$250 : : \$540 : \$135, C's share of gain.

NOTE.—In this example, \$1000 gains \$540, or, what is the same thing, each dollar of the capital gains 54 cents (the 1000th part of \$540). A has \$400, each of which gains 54 cents; his whole gain must therefore be 54 cts. × 400 = \$216 as above. By the rule, we actually do what is done in this analysis—we divide by the whole capital and multiply by each partner's share of it.

EXERCISE 64.

1. A and B enter into partnership; A pays $420 and B $280 of the capital; they gain $66; what is each man's share of the gain?
2. A, B, and C build a vessel at a cost of $50,000, of which A pays $15,000, B $25,000, and C the remainder—they lose $1,800; how much of this does each pay?
3. A merchant owes A $30, B $70, C $90, and D $120; his whole assets amount to $96; how much must he pay to each?
4. A, B, and C make up a capital of $20,000; B and C each contribute twice as much as A; but A is to receive one-third of the profits for extra services; at the end of the year they have gained $4000; what is each to receive?
5. A, B, and C agree to build a railroad, and contribute $18,000 of capital, of which B pays 2 dollars, and C 3 dollars, as often as A pays 1 dollar; they lose $2,400 by the operation; what is the loss of each?
6. Four partners, A, B, C, and D, shipped 640 sheep, of which A owned 120; B 80; C 200; and D the remainder. In a storm 160 of them were thrown overboard; how many sheep did D own, and how much was each partner's loss?
7. A, B, C, and D, hired a pasture for $120; A put in 120 sheep; B 160; C 180; and D 140; how much ought each to pay?
8. A, B, and C, purchase a farm for $3500; of which A furnished $1,500; B $1,500; and C $500. They receive $280 rent for the farm; how much of this rent should each receive?
9. A captain, mate, and 12 sailors, took a prize of $2240; of which the captain takes 14 shares; the mate 6 shares; and the remainder is equally divided among the sailors; how much did each receive?
10. A, B, and C, form a joint stock of $30,000 of which A pays $14,400; B pays $12,000, and C pays the rest; the gain for the year is $6,716.80; how much

of this must each receive, a clear salary of $2,047.50 per annum being allowed to C for his services as acting partner?

COMPOUND PARTNERSHIP.

When the stock of the several partners of a company is invested for different periods of time, we are able to distribute the gains or the losses equitably among them by the rule of Compound Partnership.

Questions in Compound Partnership are solved by the following

RULE.—*Multiply the stock of each partner by the time it is in trade, then as the sum of the products is to each product, so is the whole gain or loss to the gain or loss of each partner.*

EXAMPLE 1.—Three partners, A, B, and C, put money into trade as follows:—A put in $100 for 4 months; B $300 for 2 months, and C $500 for 3 months. They gained $250; how much is each man's share of the gain?

$$\begin{aligned} \text{A's } \$100 \times 4 &= \$400 \\ \text{B's } \$300 \times 2 &= \$600 \\ \text{C's } \$500 \times 3 &= \$1500 \end{aligned}$$

$2500 = sum of the products.

Then, $\$2500 : \$400 :: \$250$: A's share, or $\dfrac{\$400 \times 250}{2500} =$ $40.00, A's share.

$\$2500 : \$600 :: \$250$: B's share, or $\dfrac{\$600 \times 250}{2500} =$ $60.00, B's share.

$\$2500 : \$1500 :: \$250$: C's share, or $\dfrac{\$1500 \times 250}{2500} =$ $150.00, C's share.

It is evident that $100 for 4 months is equal to 4 times $100 for 1 month, and $300 for 2 months equal to 2 times

\$300 for 1 month. In the above operation, therefore, we simply reduce each partner's stock to its value for one month, and then proceed as in Simple Fellowship.

EXERCISE 65.

1. A and B enter into partnership; A puts in \$840 for 4 months, and B \$650 for 6 months; they gained \$363; what is each one's share?
2. A commenced business on January 1st, with a capital of \$3400; on April 1st, he took B into partnership, with a capital of \$2600; at the end of the year the gain was \$750; what is each one's share?
3. A, B, and C, hire a pasture for \$180; A puts in 8 cows for 10 weeks; B 20 cows for 5 weeks; and C 30 cows for 9 weeks; how much ought each to pay?
4. To a certain school A sends 2 scholars 25 days; B sends 8 scholars 15 days; C sends 5 scholars 14 days; and D sends 4 scholars 25 days. They have to pay a bill of \$10.20. How much is each man's bill?
5. A, with a capital of \$500, began trade Jan. 1, 1846, and, meeting with success, took in B as a partner, with a capital of \$600, on the first of March following; four months after, they admit C as a partner, who brought \$800 stock; at the close of the year they find the gain to be \$700; how must it be divided among the partners?
6. A and B enter into partnership; A puts in \$100 6 months, and then puts in \$50 more; B puts in \$200 4 months, and then takes out \$80; at the close of the year they find that they have gained \$95; what is the profit of each?
7. The firm A, B, and C, lost \$246; A had put in \$85 for 8 months, B \$250 for 6 months, and C \$500 for 4 months; what is each man's share of the loss?
8. A and B engaged in an adventure of \$800; A continued his stock for 6 months and received \$54 gain; B continued his 4 months and received \$40 gain; what sum did each contribute?

SECTION V.

PRACTICE.

Practice is a short method of finding the cost of any quantity when the price of a unit is given.

An Aliquot part of a quantity is such a part as, when taken a certain number of times will *exactly* make that quantity.

TABLE OF ALIQUOT PARTS.

Parts of a Dollar.	Parts of a Pound.	Parts of a Shilling.	Weight.
50 cts. = $\$\frac{1}{2}$	10s = £$\frac{1}{2}$	6d = $\frac{1}{2}$ sh.	2 qr. = $\frac{1}{2}$ cwt.
$33\frac{1}{3}$ cts. = $\$\frac{1}{3}$	6s 8d = £$\frac{1}{3}$	4d = $\frac{1}{3}$ sh.	1 qr. = $\frac{1}{4}$ cwt.
25 cts. = $\$\frac{1}{4}$	5s = £$\frac{1}{4}$	3d = $\frac{1}{4}$ sh.	16 lb.* = $\frac{1}{7}$ cwt.
20 cts. = $\$\frac{1}{5}$	4s = £$\frac{1}{5}$	2d = $\frac{1}{6}$ sh.	14 lb. = $\frac{1}{8}$ cwt.
$16\frac{2}{3}$ cts. = $\$\frac{1}{6}$	3s 4d = £$\frac{1}{6}$	$1\frac{1}{2}$d = $\frac{1}{8}$ sh.	8 lb. = $\frac{1}{14}$ cwt.
$12\frac{1}{2}$ cts. = $\$\frac{1}{8}$	2s 6d = £$\frac{1}{8}$	1d = $\frac{1}{12}$ sh.	7 lb. = $\frac{1}{16}$ cwt.
$6\frac{1}{4}$ cts. = $\$\frac{1}{16}$	2s = £$\frac{1}{10}$	$\frac{1}{2}$d = $\frac{1}{24}$ sh.	14 lb. = $\frac{1}{2}$ qr.
5 cts. = $\$\frac{1}{20}$	1s 8d = £$\frac{1}{12}$		7 lb. = $\frac{1}{4}$ qr.

* That is taking 28 lbs. for a quarter.

In solving questions, the pupil may, (in order to find out the aliquot part that he can most advantageously use), divide £1, \$1, 1 acre, &c., (as the case may be), by 2, 3, 4, &c., and select from the quotients, that are free from fractions, the numbers that will suit the case.

Example 1.—What is the price of 78 cwt. of sugar at £3 per cwt.?

78 cwt. @ £3 per cwt.

£78 = price of 78 cwt. @ £1 per cwt.
3

£234 = price of 78 cwt. @ £3 per cwt.

Here the price of 78 cwt. at £1 is evidently £78, and at £3, three times that amount. Hence, *when the price of each article is of one denomination,*

RULE.—*Multiply the price of each article by the number of articles.*

But if the price of each article is an aliquot part of a higher denomination, take the same part of the number of articles, and the result is the price in the higher denomination.

EXAMPLE 2.—What cost 60 lbs. of tea at 6s 8d. per lb.?

6s 8d. =	$\frac{1}{3}$ £	£60 =	price of 60 lb. @ £1 per lb.
		£20 =	price of 60 lb. @ 6s 8d per lb.

As 60 lbs. at £1 cost £60, at 6s. 8d. or $\frac{1}{3}$ of that rate, it must cost just $\frac{1}{3}$ of that amount.

But in very many instances the price of each article is not an aliquot part of any higher denomination, when it must be divided into parts, and the sum of the results, at the rate of each part separately, will be the price required.

EXAMPLE 3.—What cost 796 cows at £9 12s. 6d. each?

	£ 796	= price of 796 cows at				£1	0	0	each.
	9								
	£7164	=	"	"	"	£9	0	0	each.
10s. = $\frac{1}{2}$ of £1.	398	=	"	"	"	0	10	0	each.
2s 6d. = $\frac{1}{4}$ of 10s.	99 10	=	"	"	"	0	2	6	each.
	£7661 10	=	"	"	"	£9	12	6	

Sometimes the number of articles, (as well as the price of each), is expressed in different denominations; in which case, we proceed as follows:

EXAMPLE 4.—What cost 198 cwt. 2 qrs. 21 lbs. at £2 4s. 4d. per cwt?

	198 cwt. 2 qrs. 21 lbs. at £2 4s. 4d. per cwt.			
	2			
4s. = $\frac{1}{10}$ of £2.	£396	0	0	= value of 198 cwt. at £2 p. cwt.
4d. = $\frac{1}{12}$ of 4s.	39	12	0	= " " " 4s. "
2 qr. = $\frac{1}{2}$ of cwt.	3	6	0	= " " " 4d. "
14 lb. = $\frac{1}{4}$ of 2 qr.	1	2	2	= value of 2 qrs. at £2 4s. 4d. p.c.
7 lb. = $\frac{1}{2}$ of 14 lb.	0	5	6½	= " 14 lbs. " " " "
	0	2	9¼	= " 7 lbs. " " " "
	£440	8	5¾	= Price required.

Or, it might have been solved more simply, thus :

	£ 2 4 4	= value of 1 cwt.
	198	
	£438 18 0	= value of 198 cwt.
2 qrs. = ½ cwt.	1 2 2	= " 2 qrs.
14 lbs. = ¼ of 2 qrs.	0 5 6½	= " 14 lbs.
7 lbs. = ½ of 14 lbs.	0 2 9¼	= " 7 lbs.
	£440 8 5¾	= Price required.

In working questions in Practice, much depends on the *ingenuity* of the pupil.

EXERCISE 64.

1. What cost 256 lbs. of tea at 5s per lb.?
2. What cost 96 cwt. of sugar at $2.90 per cwt.?
3. What cost 120 sheep at £1 10s. each?
4. What is the price of 359 gallons of wine at 7s. 7d. per gallon?
5. What is the duty on a cwt. of opium at 8s. 6d. per lb.
6. What is the price of 290 yds. 3 qrs. of cloth at £1 11s. 6d. per yard?
7. What is the price of 179 cwt. 2 qrs. 12 lbs. of currants at £2 3s. 6½d. per cwt.?
8. What cost 3 lbs. 4 oz. 5 dwt. of gold at £3 12s. 6d. p. oz.
9. What is the rent of 5 ac. 3 roods 20 phs. at £1 8s. 2d. per acre?
10. What cost 9 tons 13 cwt. of steel at £5 19s. 8d. p. ton?
11. What cost 1 yd. 3. qr. 2 nl. of linen at $1.37½ p. yd.?
12. What cost 218⅜ lbs. of sugar at 13 cents per lb.?
13. What is the price of fencing 90 miles 3 fur. 20 phs. of railway, at £37 9s. 4d. per mile?
14. What is the price of 561 ac. 2 r. 20 p. at £1 17s. 6d. per acre?
15. What is the price of 175 tons 18 cwt. 1 qr. at £38 13s. per ton?
16. The Sun's apparent motion in the ecliptic is 59′ 8″·3 per day—how much is it in 10 days 7 hs. 20 min.?
17. What is the price of 69⅝ yds at 13s. 10d. per yd.?
18. What is the price of 79¾ lbs. at £1 2s. 11d. per lb.?
19. A boy can weigh 3 cwt. 3 qrs. 15 lbs. of tea per day—how much in 10½ days?
20. What cost 96 c. yds. 18 c. ft. 56 c. in. of stone at 12½ cents per c. yard?

SECTION VI.

PER-CENTAGE, COMMISSION, BROKERAGE, INSURANCE, PROFIT AND LOSS, INTEREST, DISCOUNT, AND BARTER.

PER-CENTAGE.

The Latin words *per* and *centum*, mean "by the hundred," when, therefore, we speak of so much "per cent." we mean so much in the hundred. If 3 persons out of every hundred died in Toronto during the year, the mortality would be expressed as 3 per cent.

The rate per cent. may be conveniently expressed in decimals, to show the rate *per unit*,—for example, 5 per cent. is 5 per hundred, or ·05 per unit, (the rate per unit being of course the 100th part of the rate per hundred), so

2	per cent.	is	·02	per unit.		$3\frac{3}{4}$	per cent.	is	·0375	per unit.
3	"	is	·03	"		$6\frac{1}{8}$	"	is	·06125	"
$2\frac{1}{2}$	"	is	·025	"		$4\frac{5}{8}$	"	is	·04625	"
$4\frac{1}{4}$	"	is	·0425	"		$1\frac{2}{3}$	"	is	$\cdot01\dot{6}$	"
$\frac{1}{4}$	"	is	·0025	"		$\frac{3}{4}$	"	is	·0075	"
$\frac{1}{2}$	"	is	·005	"		$\frac{2}{5}$	"	is	·004	"

EXAMPLE 1.—I bought a house for \$800 and paid 5 per cent. down, how much did I pay?

I paid 5 per cent. down; that is \$5 on every \$100, that is the 100th part of \$5 for every dollar $= \$\frac{5}{100} = \cdot05$. Then for \$800, it would be 800 times as much as for one dollar, that is $\cdot05 \times 800 = \$40.00$. Hence the rule for finding the *per-centage* on any quantity is

Multiply the given quantity by the rate per unit expressed decimally, and the product will be the per-centage required.

EXAMPLE.—What is 6 per cent. of $618.25.

$$
\begin{array}{rl}
\$618.25 & \\
\cdot 06 & \text{rate per unit.} \\
\hline
37\cdot 0950 & \text{per-centage, or answer.}
\end{array}
$$

EXERCISE 65.

1. What is 35 per cent. of $14720 ?
2. What is $3\frac{1}{4}$ per cent. of £240 10s. ?
3. What is $\frac{1}{2}$ per cent. of $94.48 ?
4. What is $9\frac{1}{2}$ per cent. of £10 11s. 2d. ?
5. What is 3 per cent. of 10 tons 5 cwt. 2 qrs. ?
6. What is $99\frac{1}{2}$ per cent. of 60 acres 3 roods ?
7. What is $1\frac{1}{8}$ per cent. of $1 ?
8. In a school there are 340 boys and girls, 40 per cent. of the whole are girls ; how many boys are there ?
9. A merchant bought 1000 brls. of flour and sold 10 per cent. of them, afterwards 10 per cent. of what remained, and again 20 per cent. of the remainder ; how many barrels had he remaining ?
10. A gentleman's income is $5000 per year ; he spends 6 per cent. of it as personal expenses, 30 per cent. for household expenses, 38 per cent. for rent, &c., &c., and invests the balance ; how much does he invest ?
11. The deaths in England are about $1\frac{4}{5}$ per cent. per annum of the population; if the population is 18,000,000, what is the number of deaths annually ?
12. The copper mines of Lake Superior produce *ore* containing 70 per cent. of pure copper ; how much copper is there in 1260 tons of ore ?

COMMISSION, BROKERAGE, AND INSURANCE.

Commission is the sum, at a certain rate per cent., which is charged by one person for his services in buying or selling, &c., for another.

Brokerage is of the same nature, being a per-centage paid to a broker for negotiating notes, bills, buying and selling stock, &c., &c.

Insurance is a contract by which one party, (an Insurance Company), on being paid a certain premiun or per-centage by another, engages, in case of loss by fire, &c., to pay whatever amount of loss is covered by insurance.

The nature of all questions in Commission, &c., then, it is obvious, is exactly the same as that of Per-centage already explained.

To compute the Commission, Brokerage, Insurance, or any other allowance, at so much per cent. on a given sum,

RULE.—*Multiply the given sum by the rate per unit expressed decimally.*

EXAMPLE 1.—What is the commission on \$6420 at $5\frac{1}{2}$ per cent.?

$5\frac{1}{2}$ per cent $=$ ·055 per unit.

$$\begin{array}{r} \$6420 \\ \cdot055 \\ \hline 32100 \\ 32100 \\ \hline \$353\cdot100 \end{array}$$

EXAMPLE 2.—What is the amount of a broker's commission on \$62530.20 at $\frac{1}{8}$ per cent.?

$\frac{1}{8}$ per cent. $=$ ·00125 per unit.

\$62530.20 $\times$ ·00125 $=$ \$78·16275.

Sometimes, in order to secure freedom from all loss, property is insured so that in case of loss, both the value of the property and the premium paid for the "Policy," (as the written contract is called), shall be made good to the owner. A person insuring \$100 at 5 per cent., in case of loss, receives \$100, but he has evidently lost the \$5 paid as premium. In order, therefore, to find how much must be insured on property worth a given sum, so that in case of loss, both the value of the property and the *premium* may be repaid,

RULE.—*Subtract the rate from \$100, then the remainder is to the value of the property as 100 is to the sum to be insured.*

EXAMPLE 3.—How much must be insured at 6 per cent. on \$120, so that in case of loss, not only the value of the property, but also the *premium* may be repaid?

OPERATION.

$$100 - 6 = 94.$$

Then $94 : 120 :: 100 :$ *Ans.*

\$$127.65\frac{45}{47}$. *Ans.*

The premium on \$$127.65\frac{45}{47}$, at 6 per cent. is \$$7.65\frac{45}{47}$, so that in case of loss, the owner is paid the premium as well as the value of his property.

EXERCISE 66.

1. What is the commission on £90 12s. 3d. at 8 per cent?
2. What is the premium of insurance on \$1000 at $1\frac{5}{8}$ per cent.?
3. What is the brokerage on \$99,999.80 at $\frac{3}{16}$ per cent.?
4. What is the commission on £942 16s. 3d. at $4\frac{1}{2}$ per ct.?
5. What is the commission on £946 18s. 10d. at $\frac{11}{40}$ per ct.?
6. What is the premium of insurance on £1486 13s. 9d. at $2\frac{5}{8}$ per cent.?
7. What must be insured at $5\frac{3}{4}$ per cent. on £1938 12s. 6d., so that in case of loss, both the value of goods and the premium may be repaid?
8. What is the commission on \$90 at $17\frac{1}{8}$ per cent.?
9. A broker sells stock to the value of \$87,634.75, what is his commission at $\frac{5}{8}$ per cent.?
10. Add to \$250 the commission on itself at $8\frac{1}{2}$ per cent., and find the insurance of the sum at 4 per cent?
11. What is the brokerage on \$7450 at $2\frac{1}{2}$ per cent.?
12. A factor sells 75 bales of cotton at \$450 per bale, and is to receive 2 per cent. commission; how much money must he pay to his principal?
13. An agent bought goods amounting to \$2465; what is his commission at $2\frac{1}{2}$ per cent.?
14. A gentleman paid a broker $\frac{1}{4}$ per cent. to invest \$8450 in United States stocks; how much was the brokerage?

15. An attorney collected a debt of \$3476.50, and charged $7\frac{1}{2}$ per cent. commission; how much did he receive?

16. A commission merchant sold goods to the amount of \$4536 at $2\frac{1}{2}$ per cent.; what was his commission?

17. What is the commission for selling dry goods at 4 per cent. to the amount of \$746, and groceries at 3 per cent. to the amount of \$542?

18. A commission merchant sells \$436 worth of dry goods at 3 per cent., and \$458 worth of paper at $2\frac{1}{2}$ per cent.; what is his commission?

PROFIT AND LOSS.

Such questions in per-centage as the following, are usually placed under the head of Profit and Loss.

EXAMPLE 1.—If I buy cloth at 60 cents per yard, and sell it at 75 cents; what do I gain per cent.?

ANALYSIS.—If I gain 15 cents on 60 cents, (which I do by giving 60 cents and receiving 75), then I gain $\frac{15}{60}$, *i. e.* 25 per cent.; hence,

To find the gain or loss per cent., when the buying and selling price are given:—

RULE.—*Make the gain or loss the numerator, and the buying price the denominator, then reduce the fraction to the decimal of hundredths.*

EXAMPLE 2.—If I buy at 60 cents, and in selling, gain 25 per cent.; what is the selling price?

ANALYSIS.—I paid 60 cents, and gained 25 per cent. of 60 cents, *i. e.* 15 cents; hence, $60 + 15 = 75$ cents selling price; hence,

When the *buying price* and gain or loss per cent. are given to find the *selling price*:—

RULE.—*Find the per-centage on the given cost, at the given rate per cent., and if gain, add it to, or if loss, subtract it from the cost.*

EXAMPLE 3.—If I sell at 75 cents, and gain 25 per cent.; what was the cost price?

ANALYSIS.—·0125 cents of selling price is equal to ·01 cent of buying price; hence,

When the *selling price* and the gain or loss per cent. are given to find the *buying price*:—

RULE.—*Divide the selling price by* 1 *plus the gain per unit, or, (if loss be given), by* 1 *minus the loss per unit, expressed decimally, and the quotient will be the buying price.*

EXAMPLE.—$·75 \div 1·25 = ·60$.

EXERCISE 71.

1. Bought iron for \$980; for how much must it be sold to gain 12 per cent.?
2. Bought 50 tons of steel, at \$45 per ton; how must it be sold per ton to gain 3 per cent.?
3. By selling sugar at \$8.75 per cwt., I gain 22 per cent.; what did it cost me?
4. By selling flour at \$5.50 per barrel, I lose $8\frac{1}{3}$ per cent.; what did it cost me?
5. If I sell 140 bushels of wheat for \$350, and thereby gain 24 per cent.; for how much should I have sold it a bushel, to lose 20 per cent.?
6. If I sell 30 yards of broad-cloth for \$132, and thereby gain 10 per cent.; how ought I to sell it a yard to lose 25 per cent.?
7. Sold 15 boxes of damaged raisins for \$34.50, which was at a loss of 8 per cent.; how should I have sold them a box to have gained $33\frac{1}{3}$ per cent.?
8. A merchant sold two boxes of goods for \$60 a-piece; on one he gained 20 per cent., and on the other he lost 20 per cent.; did he gain or lose by the operation, and how much?

BARTER.

The method by which two parties can exchange goods at certain prices without loss to either, is called Barter.

Thus, if I had 20 lbs. of tea worth \$9, and wished to exchange it for sugar, at the rate of \$18 per cwt.; I

ascertain by the rule of Barter what quantity of sugar I should get in exchange, viz., half a cwt.

To solve questions in Barter, the following is the

RULE.—*Find the value of the goods, the price and quantity of which are given, and divide this by the quantity of the other, and the quotient is the price; or divide by the price, and the quotient is the quantity.*

EXAMPLE 1.—How much sugar at 12 cents per lb. must I receive for 20 yards of cloth at 15 cents per yard?

$\cdot 15 \times 20 = \$3.00$ price of one, and $\$3.00 \div \cdot 12 = 25$ lbs., quantity of sugar.

EXERCISE 72.

1. A merchant exchanged 50 lbs. of tea at 45 cents per lb., for wheat at 25 cents a bushel; how many bushels did he receive?
2. A gave B 60 volumes of books at 75 cents a volume, and received in return 15 lbs. of butter at 13 cents per lb., 40 lbs. of sugar at 11 cents per lb., and the balance in tea at 4 cents per lb.; how many lbs. did he receive?
3. A gentleman wishes to exchange a farm of 80 acres, worth $50 an acre, for wild land, worth $5 an acre; how many acres did he receive?
4. A gave B 3 sheep at $2.25 each, and 11 cows at $13 each; how many bushels of corn must he receive in return at 43 cents a bushel?
5. How many horses worth $65 each, can I procure for a farm of 200 acres 2 roods, worth $17 per acre?

INTEREST.

Interest is the sum which a person pays for the use of money, and it is usually reckoned at so much per cent. per annum.

The money lent is called the *Principal*.

The sum paid for every hundred dollars of the Principal is the *rate per cent.*

The Interest and the Principal added together is called the *Amount.*

Thus, if I borrow $600 dollars for a year, and agree to pay at the rate of $6 per cent. for its use at the end of the year, I owe not only $600, but also $36 for interest. In this example we have,

$600	as the	Principal.
$6	"	Rate per cent.
$36	"	Interest.
$636	"	Amount (Principal + Interest).

When Interest is charged on the *original Principal only,* it is termed *Simple Interest.*

But when, at the end of a stated period, the interest then due is added to the principal, and this amount becomes the new principal, and so on, at the expiration of each period, instead of being paid, this is called Compound Interest.

SIMPLE INTEREST.

To find the *Interest* of a given sum for one year or any number of years,

RULE.—*Multiply the principal by the rate per unit, and the product by the time.*

EXAMPLE.—Required the interest of $240 for 3 years at 5 per cent. per annum?

$240	= principal.
·05	= rate per unit.
$12.00	= interest for 1 year.
3	
$36.00	= interest for 3 years.

When the *time* is not an *even number* of years, but includes months and weeks, &c., it will generally be found most convenient to get the interest for the months, days, &c., by Practice.

For example, if the question is, what is the interest of

of $400 for 2 years, 8 months, and 2 weeks, at 4 per cent. per annum ?

	\$400	= principal.
	·04	
6 months = ½ year.	\$16.00	= interest for 1 year.
	2	
	\$32.00	= " " 2 years.
2 months = ⅓ 6 months.	8.00	= " " 6 mos.
2 weeks = ¼ of 2 months.	2.66⅔	= " " 2 mos.
	·66⅔	= " " 2 weeks
	\$43.33⅓	

But when *aliquot parts* cannot be conveniently got for the required time, the following method may be adopted, especially as it is seldom that the *time* can conveniently be taken as aliquot parts of a year.

RULE.—*Find the interest for one year; express the given parts of a year as days, and find by proportion the interest for that time, and add it to the interest for the even number of years.*

EXAMPLE.—What is the interest of $300 for 2 years and 90 days at 6 per cent. ?

```
$300
 ·06
$18.00 = interest for 1 year.
     2
$36.00 = interest for 2 years.
```

Then to find the interest for 90 days,

```
365 : 90 : : $18.00
        18.00
365)1620.00($ 4.43 = interest for 90 days.
    1460     $36.00 =    "    "  2 years.
     1600    $40.43 = Ans.
     1460
      1400
      1095
       305
```

EXERCISE 67.

1. What is the interest of $908.20 for 1 year at 3 pr. ct.?
2. What is the interest of $624 for 2 years at 8 pr. cent.?
3. What is the amount of $400 for 4 years at 5 per ct.?
4. What is the interest of $643.70 for 6 years at 8 per cent.?
5. What is the interest of $2124.84 for 9 years at $4\frac{1}{2}$ per cent.?
6. What is the interest of $5347.62 for 5 years at 6 per cent.
7. What is the interest of $3217.68 for 11 years at 5 per cent.?
8. What is the interest of $8922.50 for 7 years at $3\frac{1}{2}$ per cent.?
9. What is the interest of $4159.71 for 13 years at 6 per cent.?
10. What is the interest of $9754.32 for 8 years at 7 per cent.?
11. What is the interest of $9112.27 for 12 years at 8 per cent.?
12. What is the interest of $3765.38 for 10 years at $7\frac{1}{2}$ per cent.?
13. What is the interest of $80 for 12 years 3 months and 20 days at 10 per cent.?
14. What is the interest of $1000 from March 1 to Jan. 9, at $9\frac{1}{4}$ per cent.?
15. What is the interest of £584 18s. 8d. for 1 year and 9 months at $3\frac{1}{8}$ per cent.?
16. What is the amount of $326 for 2 years 5 months at $4\frac{3}{4}$ per cent.?
17. What is the interest of $162 from Aug. 24 to Jan. 1, at 6 per cent?
18. What is the interest of $1 for 80 years 6 months at 15 per cent.?
19. What is the amount of $2800 for 89 days at 5 per ct.?

There are five things in Interest—the *Principal*, the *Rate per cent.*, the *Time*, the *Interest*, and the *Amount*—any *three* of which being given, the other two may be found. In most questions, the *principal*, *rate per cent.*, and *time* are given to find the *interest*, as in the last exercise; but we

may sometimes require to find the principal, rate per cent., or time.

To find the *principal* when the rate per cent., time, and interest are given,

RULE.—*Divide the given interest by the interest on one dollar, at the given rate and time; the quotient will be the required principal.*

EXAMPLE.—What sum will produce \$127.40 in 4 years at $3\frac{1}{2}$ per cent.?

\$1 at $3\frac{1}{2}$ per cent., for 4 years = ·14, and

$$\frac{127.40}{\cdot 14} = \$910, \text{ required principal.}$$

To find the *time*, when the principal, interest, and rate are given,

RULE.—*Divide the given interest by the interest on the given principal at the given rate for one year, and the quotient will be the required time in years.*

EXAMPLE.—In what time will \$240 give \$72 interest at 5 per cent.?

The interest on \$240 at 5 per cent. for 1 year = \$12, and

$$\frac{\$72 \text{ given interest.}}{\$12 \text{ interest on principal for 1 year.}} = 6 \text{ years.}$$

To find the *rate*, the principal, time, and interest being given,

RULE.—*Divide the given interest by the interest on the given principal at 1 per cent. for the given time, and the quotient will be the required rate per cent.*

EXAMPLE.—At what rate per cent. per annum will \$248 in 7 years give \$86.80 interest?

The interest on the given principal at 1 per cent. for 7 ears = \$17.36.

$$\frac{\$86.80 \text{ given interest.}}{\$17.36 \text{ interest on given principal at 1 per cent. for 7 years.}} = 5 \text{ per cent. required rate.}$$

EXERCISE 68.

1. What sum will give $25.98 of interest in 7 months at 6 per cent. per annum?
2. In what time will $893.56 give $44.68 interest at 6 per cent.?
3. At what *rate* will $856.84 in 4 years 9 weeks and 12 days give $204.93 interest?
4. A gentleman lent $2000 and received for interest $675; how long had it been unpaid, computing interest at 7 per cent. per annum?
5. I lent money for 6 years at 5 per cent., and received as *amount* due at end of that time $1040; how much did I lend at first?
6. Required the rate per cent. at which $100 will amount to $200 in 10 years?

COMPOUND INTEREST.

Money is lent at Compound Interest when interest is charged not only on the original principal, but also on the *interest* that remains unpaid.

FOR EXAMPLE.—If I borrow $100 at 5 per cent. per annum, at the end of a year I owe $105—the $100 borrowed and $5 for interest; if I do not pay the $5, at the end of the second year, I owe $105, and interest *on it* for a year, or $5.25, in all $110.25—interest being charged not only on the principal, but also on the $5 of unpaid interest.

To compute the Compound Interest of a given sum for any number of payments,

RULE.—*Find the interest of the given sum for the first period, and add it to the principal. Take this sum as the principal for the next period and find the interest thereon, and add to it the principal used for that period; proceed in this manner with each period of the given time, and the last result will be the amount of the principal for the given time, from which, of course, if the principal be subtracted, the remainder is the interest for the proposed time.*

EXAMPLE.—What is the compound interest of $960 for 3 years at 5 per cent. per annum?

	$960	= principal for first year.
Add	48	= interest " " " at 5 pr. ct.
	$1008	= principal for second year.
Add	50.40	= interest " " "
	$1058.40	= principal for third year.
Add	52.92	= interest " " "
	$1111.32	= amount at end of third year.
Subtract	$960.00	=
	$151.32	*Ans.*

When the number of periods are not numerous, this method can be used without much labour; but, when the periods are numerous, the labour becomes very great. The following plan in such cases, will be found more convenient:—

Find the amount of $1 for one period at the given rate per cent. and multiply it by itself one time less than the number of periods, then multiply the product by the number of dollars in the given principal, and from the result subtract the given principal.

EXAMPLE.—Required the Compound Interest of $600 for 5 years at 5 per cent. per annum.

Amount of $1 at 5 per cent. is $1.05 × 1.05 × 1.05 × 1.05 × 1.05 = $1.176 amount of $1.

	600	
	$765.600	amount of $600.
Subtract	600	
	$165.600	*Ans.*

The *reason* of this appears from the following analysis: Taking the example just given, the principal for the first period is $1, for the second period it is $1.05, (being the amount of $1 for that time). To find the amount of $1.05 for a period when the amount of $1 is given, we have the proportion.

$1 : $1.05 : : $1.05 : amount at end of the second period.

$$\text{or, } \frac{\$1.05 \times 1.05}{1} = 1.05 \times 1.05 = \$1.1025.$$

Again, to find amount at the end of the third period, the principal being now $\$1.05 \times \$1.05$, we have

$$\$1 : \$1.05 :: (1.05 \times 1.05) : \text{amount of third period.}$$

$$\text{or, } \frac{1.05 \times 1.05 \times 1.05}{1} = \$1.05 \times 1.05 \times 1.05 = \text{"}$$

By proceeding in the same manner, we find that the amount of \$1 for any number of periods, is equal to the amount of the first period multiplied by itself *one time less* than the number of periods.

EXERCISE 69.

Find the amounts of the following sums at the given rates per cent.

1. \$500 for 3 years at 7 per cent. per annum?
2. \$840 for 3 years at 5 per cent. per annum?
3. \$880 for 6 years at 2 per cent. per annum?
4. \$3056 for 2 years at 6 per cent. half yearly?
5. \$3 for 10 years at $4\frac{1}{2}$ per cent. per annum?
6. What is the interest of \$2615.20 for 6 years at $3\frac{1}{2}$ per cent. per annum?
7. What is the interest of \$900 for $2\frac{1}{2}$ years at 6 per cent. half yearly?
8. What is the interest of \$100 for 3 years at 5 per cent. quarterly?
9. What is the interest of \$818 for $7\frac{1}{2}$ years at 4 per cent. half yearly?
10. What is the amount of \$2329.50 for 5 years at $5\frac{1}{2}$ per cent. per annum?
11. What is the compound interest on \$200 for 8 years at 6 per cent., allowing interest to be due annually?
12. What is the compound interest on \$1000, for 2 years, at 8 per cent., allowing interest to be due half yearly?
13. What would \$500 amount to in 5 years, at 6 per cent. interest, if the interest be *compounded* annually?

DISCOUNT.

Discount is an allowance made for the payment of money before it is due.

The money received as the payment of a Bill, Mortgage, &c., due some time after, is called its *present worth.*

Thus, if I hold a note for $100 due in 6 months, but wish to obtain its value at once, a banker, on being satisfied that the note is good, gives me $95, its present worth at 10 per cent. per annum, retaining $5 as discount. The *true* present worth of a note or other security is the sum which at so much per cent. would amount to the sum named in the note in the time which is to run before the note becomes due; thus, if interest is at 5 per cent., $100 in one year would amount to $105, so that the value of a note for $105, due one year hence, is exactly $100, and the *true* discount would be $5.

But bankers in discounting, charge the interest on the sum named in the note; in the case above, they would charge as discount 5 per cent. on $105—$5.25, instead of $5, the true discount.

To compute the *true* discount on a note, &c., &c.,

RULE.—*Find the amount of* $1 *at the given rate for the given time; divide the sum for which the note, &c., is drawn by this amount, and subtract the quotient from the dividend.*

EXAMPLE.—What is the discount on a note due one year hence for $127.05 at 5 per cent.?

For one year at 5 per cent., the amount of $1 is $1.05, therefore, by the rule, $127.05 ÷ 1.05 = $121, true present worth, and $127.05 — $121 = $6.05 discount. The amount of $1 for the time, being $1.05; had the bill been drawn for $1.05, of course its present worth would be $1, but it is drawn for $127.05. Therefore,

$1.05 : $127.05 : : $1 present worth, or

$$\frac{\$127.05 \times 1}{1.05} = \frac{\$127.05}{1.05} = \$121, \text{ present worth.}$$

The *Bank* discount on any sum is just the interest on it for the time mentioned before it is due; but on all bills

3 *days of grace* are allowed, which bankers always add in computing discount. A bill dated 1st January, and drawn at 3 months, is due on 4th April, not on the first.

Example.—What is the bank discount on a note for $120, due in 6 months at 5 per cent?

Interest of $120 for 6 mo. at 5 p. ct. per annum is	$3.00
Add interest of " 3 days " " "	·049
Bank discount is	$3.049

EXERCISE 70.

1. What is the bank discount on a bill for $2196 for 32 days at 5 per cent. per annnm?
2. What is the true discount on a note for $370 for 3 months at 7 per cent. per annum?
3. What is the *true* and the *bank* discount on $99.20 for 1 year at $5\frac{1}{2}$ per cent.?
4. What is the *true* present worth of $139.81, due 10 years hence, at $5\frac{1}{2}$ per cent.?
5. In the last example, reckoning *bank* manner of discount, what is the present worth? How much more than the true discount does the banker charge?
6. I discounted at the bank the following bills, at the rate of 8 per cent. per annum: one note for $1000 due in 6 months, two notes for $500 each, due in 70 days, and one for $320, due in 2 years; how much did I receive in all?
7. What is the discount on a note for $911.40, drawn at 5 months, at 8 per cent.?
8. What is the discount on a note, dated March 26th, and due June 23rd, for $897.20, at $3\frac{3}{4}$ per cent.?
9. A note of $1651.50 is due in 11 months, but the person to whom it is payable sells it with the discount off at 6 per cent.; how much shall he receive?
10. C owes D $3456, to be paid October 27th, 1842; C. wishes to pay on the 24th of August, 1838, to which D consents; how much ought D to receive, interest at 6 per cent.?

SECTION V.

INVOLUTION AND EVOLUTION.

When we multiply any number *by itself* any number of times, the several products are called *powers* of the number; thus if 4 is multiplied by itself once, the product is 16, a *power of four*, if multiplied twice, ($4 \times 4 \times 4 = 64$), the product 64 is another *power* of four.

The number which by multiplication produces the power, is called the *root*.

In the example taken above, 4 is the *root* of 16 and 64.

The *powers* are called *first, second, third, &c.,* according to the number of times the *root* is taken as factor; thus, *first* power of 4 is 4, *second* power is 16, *third* power is 64, and so on.

The *second* power is also called the *square*, and the third power the *cube*. Powers are often indicated by writing after the number, and a little higher the number which shows the number of times it is taken as a *factor*; thus, 4^2 = second power of 4; 7^6 = sixth power of 7; this small figure is called the *index* or *exponent*.

The process of finding a *power* of a number is called *Involution*, and the process of finding any *root* of a number is called *Evolution*.

INVOLUTION.

To involve a number to any required power,

RULE.—*Take the number as a factor, as often as is indicated by the index of the power, the continued product of these factors is the required power.*

EXAMPLE 1.—What is the 4th power of 5.

$$5^4 = 5 \times 5 \times 5 \times 5 = 625 \ \textit{Ans.}$$

EXAMPLE 2.—What is the 3rd power of $3\frac{1}{2}$.

$$3\tfrac{1}{2} = \tfrac{7}{2} \text{ and } \tfrac{7}{2} \times \tfrac{7}{2} \times \tfrac{7}{2} = \tfrac{343}{8} = 42{\cdot}875 \ \textit{Ans.}$$

To involve mixed numbers, we reduce them first to improper fractions.

EXERCISE 74.

Involve,

1. 8 to the fourth power.
2. 23 to the fifth power.
3. 225 to the tenth power.
4. $9\frac{1}{2}$ to the fourth power.
5. $\frac{4}{5}$ to the third power.

Involve,

6. 1·25 to the second power.
7. ·901 to the third power.
8. 1·1 to the sixth power.
9. $3\frac{2\frac{1}{2}}{9}$ to the second power.

EVOLUTION.

EXTRACTION OF THE SQUARE ROOT.

To extract the *square root* of a number, is to find a number which if multiplied *once* by itself would produce the number.

The theory of the rules for the extraction of the Square and the Cube Root, can be best explained by the teacher, with blocks made for the purpose, and this should be thoroughly done in the outset.

Before the pupil proceeds to the exercises, the question, "why?" should be put and answered in reference to each step in the rule.

RULE.

(1.)—*Point off the given number into periods of two figures each, (pointing from right to the left hand in whole numbers, and from left to right in decimals.)*

(2.) *Find the highest square contained in the left hand period, and place its root as a quotient in division.*

(3.) *Subtract the square number from the left hand period, and to the remainder, if any, bring down the next period for a dividend.*

(4.) *Double the root already found for a trial divisor.*

(5.) *Find how often the trial divisor is contained in the dividend, omitting the last figure, and place the figure thus obtained in the root, and also to the right of the trial divisor.*

(6.) *Multiply the divisor by the figure placed last in the root; subtract the product from the dividend, and to the remainder, if any, bring down the next period for a new dividend.*

(7.) *Double the part of the root now found for a new trial divisor, and proceed as before, continuing the process until all the periods are brought down.*

NOTE.—In extracting the square root of fractions, if the numerator and denominator are both complete squares, extract the square root of them separately, otherwise reduce it to a decimal fraction and proceed according to the rule.

EXAMPLE.—What is the square root of 2025?

```
20          2025(45 the root.
 4 × 4 =    16
     2      ---
    --      425
    85      425
```

We first point off from the right into periods of two figures each, and find the root must contain two digits; the left hand period is 20; the highest square in 20 is 16, the square of 4; we place 4 in the root as the first figure, and subtract 16, its square, from 20, and bring down the next period; we then double the 4 for a trial divisor, and find how often 8 is contained in 42, which is 5 times; we put the 5 in the root, and also to the right of the trial divisior, and multiplying 5, find the work completed.

EXERCISE 73.

What is the square root of the following numbers :—

1. 1316, 222784.
2. 4922894, 5499025.
3. 67305616, 5.
4. ·5, [illegible].
5. $\frac{9}{16}$, $\frac{4}{25}$.
6. $\frac{16}{100}$, $\frac{81}{144}$.
7. $20\frac{1}{4}$. $1\frac{[illegible]}{[illegible]}$.
8. $207\frac{25}{36}$, 338·683604.
9. [illegible], [illegible].
10. $23784\frac{[illegible]}{100}$, $74786\frac{[illegible]}{1000}$.
11. 90374376, 28478783.
12. $847376\frac{[illegible]}{[illegible]}$, $783703\frac{[illegible]}{[illegible]}$.

EXTRACTION OF THE CUBE ROOT.

To extract the cube root of a number, is to find a root which when multiplied twice by itself, produces the number; thus, the cube root of 27 is 3, for $3 \times 3 \times 3 = 27$.

The teacher should explain the rule for the extraction of the cube root, by using the blocks, before the pupil is required to commit or use it; and the pupil should be able to answer the question, "why," in reference to each step in the rule, before he proceeds to the exercise.

RULE.

(1.) *Point off the given number into periods of three figures each, pointing to the left in whole numbers, and to the right in decimals.*

(2.) *Find the highest cube contained in the left hand period and place its root as a quotient in division.*

(3.) *Subtract the cube from the left hand period, and to the remainder, if any, bring down the next period for a new dividend.*

(4.) *Multiply the square of the part of the root found by 300, for a trial divisor.*

(5.) *Find how often this trial divisor is contained in the dividend, and put the figure thus obtained in the root.*

(6.) *Complete the trial divisor by adding to it the product of the root previously found, multiplied by the last digit of the root, and that product multiplied by 30, and also the square of the last digit in the root.*

(7.) *Multiply the divisor thus completed, by the last digit in the root, and subtract the product from the dividend, and bring down the next period for a new divide. d.*

(8.) *Multiply the square of the part of the root now found by 300, for a new trial divisor, and proceed as before until all the periods are brought down.*

In fractions, when both the numerator and denominator are complete cubes, extract the cube root of each separately; in all other cases reduce the fraction to a decimal, and proceed as in whole numbers.

EXAMPLE.—What is the cube root of 1953125?

			1953115(125
			1
1st trial div.	$1^2 \times 300 =$	300	953 = 1st dividend.
1st increase	$1 \times 2 \times 30 =$	60	
2nd "	$2 \times 2 =$	4	
Complete divisor	=	364	728
2nd trial div.	$12^2 \times 300 =$	43200	225125 = 2nd divid.
1st increase	$12 \times 5 \times 30 =$	1800	
2nd "	$5 \times 5 =$	25	
Complete divisor	=	45025	225125

NOTE.—The following method of extracting the cube root involves much less labour than the foregoing, and is founded on the same principle.

Having pointed off the number into periods of three figures each, as in the former method, place at moderate in-

tervals before it two ciphers as the commencement of two columns; having found the first figure of the root, place it in the first column, and having added it to what stands above it, multiply the sum by the same figure, writing the product in the second column; add in like manner in the second column, multiply the sum by the same figure, and place the product in the *third column*, and subtract it from what stands above it.

Perform a process exactly similar in the first and second columns, add the figure found in the root to the first column, then add one cipher to the *first*, and two ciphers to the *second* column, and bring down a period of three figures to the *third* column; divide the third column by the second as a *trial divisor* to find the next figure in the root; place this figure in the first column, and proceed as before.

EXAMPLE.—(Same number as before.)

1st Column.	2nd Column.	3rd Column.
0	0	1953125(125
1	1	1
1	1	953
1	2	
2	300	
1	64	
30	364	728
2	68	
32	43200	225125
2	1825	
34	45025	225125
2		
360		
5		
365		

EXERCISE 75.

Extract the cube root of the following numbers :—

1. 373248, 592704.
2. 12326391, 15625.
3. 2985984, 704969.
4. $\frac{125}{216}$, 21024576.
5. 84·604519, ·000343.
6. 2, 997002999.
7. ·389017, $\frac{343}{1000}$.
8. $31\frac{15}{343}$, $\frac{250}{686}$.

APPLICATION OF THE SQUARE AND CUBE ROOT.

1. If an acre of land be laid out in a square form, what will be the length of each side in rods?
2. What will be the length of the side of a square, in rods, that shall contain 100 acres?
3. A cellar is 25 feet long, 20 feet wide, and $8\frac{1}{2}$ feet deep; what will be the dimensions of another cellar of equal capacity in the form of a cube?
4. What will be the length of one side of a cubical granary that shall contain 2500 bushels of grain?
5. What is the length, in rods, of one side of a square that shall contain 12 acres?
6. A company of speculators bought a tract of land for $6724, each agreeing to pay as many dollars as there were partners; how many partners were there?
7. What must be the length, depth, and breadth of a box, when these dimensions are all equal and the box contains 4913 cubic feet?
8. The solidity of a cubical block is 21952 cubic yards; what is the length of each side? What is the area of the surface?
9. A general has an army of 7225 men; how many must be put in each line in order to place them in a square form?
10. Two persons start from the same point; one travels due east 50 miles, the other due south 84 miles; how far are they apart?
11. How many small cubes of 2 inches on a side can be sawed out of a cube 2 feet on a side, if nothing is lost in sawing?

FINAL COMPREHENSIVE MISCELLANEOUS EXERCISE.

1. Write down as one number, five thousand billion, four thousand and three, and ninety-two thousandths.
2. Add twenty thousand three hundred and twenty-five millionths to one hundred thousand and twenty-five ten-thousandths.
3. Multiply $9873\frac{1}{4}$ by 785·18231.
4. One gentleman meeting another, and inquiring the time past 12 o'clock, received for an answer, $\frac{17}{19}$ of the time from now to midnight; what o'clock in the afternoon was it?
5. From one half of a piece of cloth containing 82 yds. 2 qrs., a tailor cut six suits of clothes; how much did each suit contain?
6. Multiply $23{\cdot}5\dot{6}\dot{4}$ by $2{\cdot}6\dot{5}$, and divide the result by $2{\cdot}1\dot{2}\dot{3} + ({\cdot}\dot{2}\dot{3} - {\cdot}006\dot{2}.)$
7. What is the exact decimal value of $\frac{5}{29}$.
8. If £100 of bank stock is worth £$98\frac{7}{8}$, what is £362 8s. $7\frac{1}{2}$d. of stock worth?
9. If you pay £37 10s. per ton for iron, at what rate must you sell it to gain the price of 1 ton on 15 tons?
10. What will be the rent of 35 acres, 2 roods, 10 r. of land, if 46 acres, 3 roods, 14 r. are worth £50?
11. If a landlord deducts $\frac{3}{8}$ on a shilling to his tenant, what will be the deduction on £76 3s. $4\frac{1}{3}$d.?
12. If $\frac{1}{3}$ and $\frac{1}{10}$ of a pasture cost £4 10s., what will the whole pasture cost?
13. Bought 840 apples, at the rate of 10 for a penny, and 240 more, at 8 for a penny; if I sell them at 36 for 4d., shall I gain or lose by the operation, and how much per cent.?
14. A farmer having sold $\frac{1}{2}$ and $\frac{1}{3}$ of his sheep, had 95 left; how many had he at first?
15. A man having \$15750, spent $\frac{1}{2}$ for a house, $\frac{1}{2}$ the remainder for a barn, and $\frac{1}{3}$ of the balance for a carriage; how much had he left?

16. What is the difference between $\frac{18}{25}$ of 275, and $\frac{3}{5}$ of 315?

17. What number is that, $\frac{1}{2}$ of which exceeds $\frac{1}{4}$ by 387?

18. What number is that, $\frac{3}{4}$ and $\frac{2}{3}$ of which make 255?

19. What number must be added to $1375\frac{1}{4}$ to make $81193\frac{3}{8}$?

20. What must be taken from $1137\frac{3}{8}$ that $793\frac{1}{4}$ may be left?

21. What must be added to $217\frac{1}{2}$ that the sum may be $17\frac{1}{4}$ times $19\frac{1}{2}$?

22. What number multiplied by $45\frac{3}{8}$, will produce $288\frac{1}{4}$?

23. What number divided by $37\frac{1}{2}$, will give $193\frac{1}{4}$ for the quotient?

24. Bought $\frac{7}{8}$ of a ship, and sold $\frac{3}{5}$ of it; how much was left?

25. A grocer used a false weight of $13\frac{1}{2}$ oz. for a pound; what was the amount of his fraud in weighing 500 lbs.?

26. If $\frac{7}{15}$ lbs. of lard cost $\frac{14}{17}$s.; how much will $\frac{33}{35}$ ton cost?

27. What number is that, $\frac{1}{2}$ of which exceeds $\frac{1}{3}$ by 428?

28. What number is that, $\frac{2}{3}$ and $\frac{3}{4}$ of which is 510?

29. A father gave his eldest son twice as much as the second, the second three times as much as the third, who had \$1573; how much did he give to all?

30. A man having 4 children, gave twice as much to the 4th as to the 3rd; twice as much to the 2nd as to the 4th; and to the 1st twice as much as to the 2nd, which was \$7860; what did he give to all?

31. A man gave $\frac{1}{3}$ of his estate to his eldest daughter; $\frac{1}{2}$ the remainder to the 2nd; and $\frac{1}{4}$ of the remainder to the 3rd, who received \$3560; what was his estate?

32. A and B travelling, A has 5 loaves of bread, and B has 3. They are overtaken by C, who says, "Let me partake with you, and I will pay for what I eat." They all eat an equal quantity, and C pays \$8. How shall

the money be divided equitably between A and B?

33. B and C together can build a boat in 18 days; with the assistance of A they can do it in 11 days; in what time would A do it himself?

34. If A can do a piece of work alone in 10 days, and A and B together in 7 days, in what time can B do it alone?

35. A, B, and C can complete a piece of work together in 12 days; C can do it alone in 24 days, and A in 34 days; in what time could B do it by himself?

36. A can do a piece of work in 3 weeks, B can do thrice as much in 8 weeks, and C 5 times as much in 12 weeks; in what time can they finish it jointly?

37. Bought 120 oranges at 2 a penny, and 120 more at 3 a penny, and sold them all together at five for 2d.; what did I gain or lose by the bargain?

38. A man left his two sons $1000; their ages were 14 and 18 years respectively; if their shares were put to interest at 6 per cent. per annum, they would be equal when each would be 21 years of age; what was the share of each?

39. What is to pay for the rent of a house, at $\$372 \cdot \dot{2}0\dot{3}$ a year, for 5 years in arrears, at 6 per cent. (simple interest?)

40. The head of a fish is 4 feet long, the tail as long as the head and $\frac{1}{2}$ the length of the body, the body as long as the head and tail; what is the length of the fish?

41. The sum of two numbers is $266\frac{2}{3}$, the product of the greater multiplied by 3, equals the product of the less multiplied by 5; what are the numbers?

42. A military officer placed his men in a square; being reinforced by three times his number, he placed the whole again in a square; again being reinforced by three times his last number, he placed the whole a third time in a square, which had 40 men on each side; how many men had he at first?

43. Suppose that a man stands 80 feet from a steeple, that a line to him from the top of the steeple is 100

feet long, and that the spire is three times as high as the steeple; what is the length of a line reaching from the top of the spire to the man?

44. Two ships sail from the same port; one sails directly east at the rate of 10 miles, the other directly south at the rate of $7\frac{1}{2}$ miles an hour; how far are they apart at the end of 3 days?

45. If eggs be bought at the rate of 5 for 2 cents; how must they be sold to gain 40 per cent.?

46. Three gentlemen contribute towards the building of a church, $657, the church is distant from the first 2 miles, from the second $2\frac{7}{8}$ miles, and from the third $3\frac{1}{2}$ miles; they agree that their shares shall be reciprocally proportional to their distances from the church; how much must each contribute?

47. If a merchant each year increase his capital by a fifth part of itself, except an expenditure of $400 per annum, and at the end of 15 years is worth $1200; what was his original capital?

48. A's note of $7851.04 was dated Sept. 5th, 1837, on which were endorsed the following payments, viz.: Nov. 13th, 1839, $416.98; May 10th, 1840, $152; what was due March 1st, 1841, the interest being 6 per cent.?

49. A person dying, worth $5460, left a wife and 2 children, a son and a daughter, absent in a foreign country. He directed that if his son returned, the mother should have one-third of the estate, and the son the remainder; but if the daughter returned, she should have one-third, and the mother the remainder. Now it so happened that they both returned; how must the estate be divided to fulfil the father's intentions?

50. If 12 apples be worth as much as 17 pears, and 2 pears cost $1\frac{1}{2}$d.; what is the value of 99 apples?

51. Place the nine natural numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, in such a manner that the sum of the odd digits shall be equal to the sum of the even ones.

52. What part of 3d. is a third part of 2d.?

53. How must the nine digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, be placed in the form of a square, so that the sum of each row of figures, reckoned upwards, downwards, horizontally, diagonally, shall just equal 15.

54. I owe my friend a shilling; he has nothing but coins worth 17s. each, and I have nothing but guineas. How must an exchange take place between us, so that I may pay my debt, but no more?

55. How must a board that is 16 inches long, and 9 inches broad, be cut, so that when the two parts are joined together, they may form a square?

56. A has by him $1\frac{1}{2}$ cwt. of tea, the prime cost of which was £96. Now, granting interest to be at 5 per cent., it is required to find how he must sell it per lb. to B, so that by taking his note, payable at 3 months, he may clear 20 guineas by the bargain.

57. The hour and minute hand of a clock are exactly together at 12 o'clock; when are they next together?

58. There is an island 73 miles in circumference, and 3 persons all start together to travel the same way about it; A goes 5 miles a day, B 8, and C 10; when will they all come together again?

59. Sold goods for 60 guineas, and by so doing lost 17 per cent., whereas I ought, in dealing, to have cleared 20 per cent.; how much were they sold under their just value?

60. A hare starts 40 yards before a greyhound, and is not perceived by him till she has been running 40 seconds; away she goes at the rate of 10 miles an hour, and the dog, on view, makes after her at the rate of 18; how long will the course hold, and what ground will be run over, beginning with the outsetting of the dog?

61. A reservoir for water has two cocks to supply it; by the first alone it may be filled in 40 minutes, and by the second in 50 minutes; it has likewise a discharging cock, by which it may, when full, be emptied in 25 minutes. Now, if these three cocks are all left open, when the water comes in, in what time

would the cistern be filled, supposing the influx and efflux of the water to be always alike?

62. A person left 40s. to 4 poor widows, A, B, C, and D, in the following proportion; to A he left $\frac{1}{3}$, to B $\frac{1}{4}$, to C $\frac{1}{5}$, to D $\frac{1}{6}$, desiring the whole might be distributed accordingly; what is the proper share of each?

63. How many oaken planks will floor a barn $60\frac{1}{2}$ feet long, and $33\frac{1}{2}$ wide; when the planks are 15 feet long, and 15 inches wide?

64. The amount of a sum of money which has been put out to interest is £100, and the principal is just 7 times as much as the interest; what is the principal?

65. A tradesman increased his estate annually $\frac{1}{3}$ part, abating £100, which he usually spent in his family; and at the end of $3\frac{1}{4}$ years, found that his net estate amounted to £3179 11s. 8d.; what had he at his outsetting?

66. A person after spending $\frac{1}{3}$ of his yearly income plus £10, had then remaining $\frac{1}{2}$ plus £15; what was his income?

67. A cistern containing 60 gallons of water has 3 unequal cocks for discharging it; the greatest cock will empty it in 1 hour; the second in 2 hours; and the third in 3; in what time will it be empty if they all run together?

68. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, and 50 of them cherries; how many trees are there in all?

69. Suppose the sea allowance for sailors to be 5 lbs. of beef, and 3 lbs. of biscuit a day, for a mess of 4 people; and that the price of the first is $2\frac{1}{4}$d. per lb., and of the second $1\frac{1}{2}$d.; now, if the ship's company be such that the meat they eat cost the government 12 guineas per day, what must they pay for their bread per week?

70. A stationer sold quills at 11s. a thousand, by which he cleared $\frac{3}{8}$ of the money; but he afterwards raised them to 13s. 6d. a thousand; what did he clear per cent. by the latter price?

FINAL MISCELLANEOUS MENTAL EXERCISE.

1. How many are 12 and 9? 11 and 17? 17 and 19?
2. How many are 28 and 43? 46 and 37? 126 and 514?
3. How many are $7 + 9 + 15 + 27 + 5 + 17$.
4. How many are $17 - 6$? $42 - 16$? $523 - 54$?
5. How many are $127 - 59$? $615 - 173$? $2986 - 175$?
6. How many are $1276 - 14 - 123 + 24 - 18 + 154$?
7. How many are $6 \times 2 - 5 + 8$? $24 \times 17 - 15 + 8 \times 36$?
8. How many are $176 \times 17 - 23 + 8$? $54 \times 26 + 19 - 23 \div 9$?
9. How many are $28 \times 15 - 11 + 18 \div 14$? $163 \times 17 - 19 + 126 \div 15$?
10. How many are $12 \times \frac{3}{4}$? $12 \div \frac{3}{4}$? $\frac{1}{6}$ of 20? $\frac{12}{13}$ of 10?
11. 4 times 8 are how many times 5, 6, and 7?
12. $2\frac{3}{4}$ of 12 are how many times $\frac{3}{4}$ of 18?
13. What is $\frac{2}{9}$ of that number of which 63 is $\frac{7}{9}$?
14. 15 is $\frac{3}{5}$ of how many times 27?
15. $\frac{2}{3}$ of 126 is $\frac{3}{4}$ of how many times $\frac{3}{4}$ of 617?
16. 17 books at \$5.27? 126 horses at \$57,96?
17. $\frac{3}{7}$ of 624? $\frac{11}{12}$ of 826? $\frac{5}{7}$ of 483?
18. How much will $\frac{5}{6}$ of a barrel of apples cost, if $\frac{3}{11}$ cos 75 cents?
19. How much will 6 cords of wood cost, if $\frac{5}{9}$ of 4 cords cost \$12.
20. $123\frac{3}{4} - 59\frac{5}{8}$? $\frac{3}{4} \times \frac{5}{6} \times \frac{17}{18}$? $\frac{6}{7} \times \frac{5}{9} \div \frac{2}{3}$ of $\frac{3}{4}$?
21. 18 is 4 times what number? 36 is $\frac{1}{5}$ of $\frac{3}{8}$ of what number?
22. If 15 cords of wood cost \$65, what will $11\frac{1}{2}$ cords cost?
23. If \$126 pay 17 men, how many will pay 150 men?
24. If 17 men can build a barn in 17 days, in how many days can 27 men build it?
25. A can do a piece of work in 8 days, B in 12 days; in what time can both working together do it?

26. I paid $\frac{3}{8}$ of a debt, and afterwards $\frac{1}{3}$ of the balance, and had \$127 left; what was the debt?

27. How much is 11 per cent. off \$125?

28. 654 lbs of cheese at $12\frac{3}{4}$ cents per lb.?

29. Bought at 15 cents and sold at 19 cents; what was the gain per cent?

30. Bought at 53 cents and sold at 15 per cent. profit; what did I sell for?

31. What is the interest on \$763, at $9\frac{3}{4}$ per cent.?

32. Interest on \$826 for 3 years, at $4\frac{1}{2}$ per cent.?

33. Bought flour at \$5 per barrel, and sold it at \$4.35; what was my loss per cent.?

34. What is the amount of \$153 for 9 months, at 4 per cent. per annum?

35. $87\frac{1}{2}$ yds. of cloth, at $57\frac{3}{4}$ cent. per $\frac{1}{4}$ yd.?

36. The interest on \$123, at 8 per cent. per annum was \$15; what was the time?

37. The interest on \$27 for 3 years was \$5.15; what was the rate per cent.?

38. The interest at 4 per cent. was \$29; what was the principal?

39. What number is that, to which if $\frac{1}{2}$ be added, the sum will be 15?

40. What number is that which being increased by its $\frac{1}{2}$, its $\frac{1}{3}$, and 18 more, will be doubled?

41. Divide the number 36 into two parts, that shall be to each other as 5 to 4.

42. Divide the number 49 into two parts, that shall be to each other as 1 to $\frac{3}{4}$.

43. A fishing rod, the length of which is 24 feet, is in two parts; $\frac{3}{5}$ of the longer part equals the length of the shorter; how long is each part?

44. $\frac{2}{3}$ of A's number of sheep + $\frac{3}{4}$ of B's number, equals 900; how many sheep has each, providing $\frac{3}{4}$ of B's number is twice $\frac{2}{3}$ of A's number?

45. Reduce 9 years and 8 months to the fraction of a year.

46. Reduce 2 years 4 months and 15 days to the fraction of a year.

47. Reduce 5 years 9 months and 18 days to the fraction of a year.

48. At 2 per cent., what part of the principal equals the interest? What part at 8, 10, 7, 5, 6, 18, 25?

49. A man sold a quantity of goods, and thereby gained 75 per cent. on the cost; what part of the cost equals the gain?

50. If $\frac{2}{50}$ of the principal equals the interest, what is the rate per cent?

51. A book was sold for $\frac{2}{3}$ of $\frac{3}{5}$ of what it cost; what was the loss per cent.?

52. How many pounds of coffee, at 8 cents a pound, must be given for $9\frac{1}{2}$ pounds of sugar, at 10 cents a pound.

53. What is the sum of 15, 5, 3, 2, 8, 3, 7, 2, 9, 6, 7, 8, 5, 6, 11, 3, 5, 14, 1, 9, 4, 8, 3, 5, 7, 9, 8, 6, 4, 2, 10, 12, 4, 9, 7, 11, 6, 10, 8, 9, 16.

54. What is the sum of 16, 4, 10, 6, 3, 7, 2, 5, 9, 11, 20, 4, 6, 8, 3, 7, 4, 3 times 5, 3, 8 less 5, 7, 4, 9, 12, 7, 3, 5, 4, 7, 8, 5, 4, 14, 8, 3, 7, 11, 5, 25, 8, 2, 4, 8, 6, 7, 8, 10, 12, 13, and 4 times 5.

55. What is the sum of 11, 5, 8, 3, 12, 4, 5, 20 less 7, 5, 8, 9, 3, 5, 13 less 5, 7, 16 less $8\frac{1}{2}$, $4\frac{1}{4}$, 7, $9\frac{1}{4}$, and 11 times 4.

56. Benjamin Franklin died in 1790, and was 84 years of age at his death; in what year was he born?

NOTE.—The earth turns on its axis from west to east, that is 360° once every 24 hours; hence it revolves 15° in one hour, that is 1° in 4 minutes, that is 1′ of distance in 4 seconds of time. As the earth turns from west to east, the farther the place is east the earlier it gets the sun, and hence, the later it is in the day.

From this we may easily tell the difference in time between any two places, if we only know the difference in longitude. To find the difference in longitude, if the places are both east or both west longitude, subtract. If one is east and the other west, add them.

57. Is it earlier or later at Toronto or Frederickton, and how much?

58. When it is 9 o'clock, a.m., at Toronto, what time is it at Frederickton?

59. When it is 3 o'clock at Kingston, what time is it at Montreal, St. John's, Halifax, New York?

60. When it is 6 o'clock, p.m., in Toronto, at what places is it mid-night, and at what places is it mid-day?

61. 7 hours less than 5 o'clock, p.m., is what o'clock? 8 hours 15 minutes less than 3 o'clock 20 minutes, is what time?

62. Reduce $\frac{2}{9}$ to 18ths, $\frac{5}{12}$ to 84ths, $\frac{3}{7}$ to 56ths.

63. How many lbs. of coffee in $\frac{1}{2}$, $\frac{5}{12}$, $\frac{3}{8}$, $\frac{5}{6}$, $\frac{3}{16}$, and $\frac{3}{4}$.

64. If I pay $37\frac{1}{2}$ cents for 4 lbs of sugar, how much must I pay for 8 pounds at the same rate?

65. $\frac{4}{5}$ of 30 is $\frac{2}{3}$ of what number?

66. If 6 men can do a piece of work in 13 days, in what time can 11 men do it?

67. If 7 men can dig a ditch in 13 days, in what time can 11 men dig one 3 times as long?

68. How many yards of carpeting that is $\frac{3}{4}$ of a yard wide, are equal to 17 yards, that is $\frac{7}{8}$ of a yd. wide?

69. If $\frac{1}{3}$ of a pole stand in the water, $\frac{1}{4}$ in the mud, and 10 feet above the water, what is the length of the pole?

70. If 12 horses consume 720 bushels of oats in 3 months, in what time will 18 horses consume 1200 bushels?

71. The distance from A to B, which is 40 miles, is $\frac{4}{5}$ of $\frac{3}{4}$ the distance from C to D; what is the distance from C to D?

72. What is the annual insurance on a dwelling house, valued at $2000, $1\frac{1}{2}$ per cent.?

73. If I buy a house for $600, and sell it again for 15 per cent. advance, how much will I sell it for?

74. What number is that, which, if it be increased by its fifth, and seventh, and 23 more, will be doubled?

75. What is the difference of time, the longitude being given, between London, England, and Washington, Boston, and Madrid.

76. When it is twenty minutes past 3 at London, what o'clock is it at each of the other places mentioned in the last question.

ANSWERS.

EXERCISE 1.

VIII, XI, XV, XIX, XXIX, XXXV, XCIX, CLX, CCCXL, DLXIX, MCVI, MMXXV, DCXCIX, MDCCCCXXV, MMDCLXXX, MMMMDCCCCLXV, MMDCCCCLXI, MCCCXL.

EXERCISE 2

74, 140, 999, 750, ·4, ·27, ·014, ·760, 76·027, 675·864, 515·06.

EXERCISE 3.

For instance, 1st, Four and six tenths, or forty-six tenths; 4·6 or $4\frac{6}{10}$ or $\frac{46}{10}$. 2nd, Sixty-three and seventy-two hundredths; or six thousand three hundred and seventy two hundredths; or, six thousand hundredths and three hundred hundredths and seventy hundredths and two hundredths; or sixty units and thirty tenths and seventy-two hundredths, &c., &c.

EXERCISE 4.

1. 22 cents.	4. 35 sheep.	7. 22 dollars.
2. 21 dollars.	5. 13 cents.	8. 15 dollars.
3. 18 marbles.	6. 95 cents.	

EXERCISE 5.

1. $1186·37.	13. $34950.10.	29. $15388.
2. 1248.	14. 21867.	30. 4258.
3. $1348.74.	15. 18068.93\frac{1}{4}$.	31. $24731.
4. 1465.	16. 10913.	32. $1658286.
5. 2250·979.	17. 30155·740.	33. $7861214.
6. 2072.	18. 18001.	34. $536146.
7. 2343·190.	19. 20170·12.	35. 75675.
8. 2856.	20. 14372.	36. 311013.
9. 975·5162.	25. $105.	37. $660.11.
10. 1635.	26. $2.93.	38. 2246 apples.
11. $1517.94.	27. $408.	39. 72 apples.
12. 1056.	28. $1475.	

EXERCISE 6.

1. 162 cents.
2. 62 cents.
3. 28½ dollars.
4. 156 times.
5. 66 years.
6. 109 trees.
7. $15.24.
8. 7 cents.
9. 8, 16, 32, 40, 64, 72.
10. 537 pounds.

EXERCISE 7.

1. 11.
2. 6 dollars.
3. 3, 11 in both.
4. 6 years.
5. $9, $21 for both
6. 15 apples.
7. 29 dollars.
8. 27 years.
9. 92 dollars.

EXERCISE 8.

1. 36919.
2. $78372.66.
3. $40253.41.
4. 38999.
5. $22984.09.
6. 15289.
7. $78359.03.
8. $5292.
9. $462121934.46.
10. $4351951.69.
11. 7392·2070.
12. 612663992.
13. 72299·5412.
14. $913109.19.
15. 457555.
16. 1205999·32.
17. ·00036.
18. 57955.
19. $8072.
20. 171.
21. 344.
22. 172.
23. 178.
24. 106.
25. 135.
26. 799.
27. 1386517.
28. 11 years.

EXERCISE 9.

1. 83.
2. 2720.
3. 1557.
4. 162.
5. 415.
6. 221.
7. 1244556.
8. $287.

EXERCISE 10.

1. 1706.
2. 59 both, 9 more.
3. $17.50.
4. 10.
5. 16.
6. $10 gain.
7. 20.
8. $466 $79.
9. $10.

EXERCISE 11.

1. 63.
2. $120.
3. 16, 20, 24.
4. John 10, Jos. 45, all 64.
5. 7146 dollars.
6. 304 yards, 2128 dollars.

EXERCISE 12.

1. $17104.64.
2. $134574.86.
3. $432266.30.
4. $225804.15.
5. 66276.
6. $672612.
7. 389304.
8. $748731·60.
9. 502557.
10. $1162129.68.
11. 574675.
12. $568669.44.
13. 350·184.
14. 6128·22.
15. 7879·14.
16. 525·276.
17. 2626.38.
18. 4377·30.
19. 8754·60.
20. 9630·06.
21. 105·0552.
22. $1966.54.
23. $6882.89.
24. $3933.08.
25. $786·616.
26. $589·962.
27. $4916.35.
28. $8849.43.
29. $11799.24.
30. $10815.97.

EXERCISE 13.

31. $68236·48.
32. $133863.66.
33. $232499.52.
34. $182214.09.
35. $231504.12.
36. $208963.44.
37. $199122.30.
38. $138250.56.
39. $565184.16.
40. $220399.92
41. $576676.32.
42. $715501.44.

EXERCISE 14.

1. $63221·592.
2. $7464·4808.
3. $2905·0420.
4. $488·44096.
5. $84393·932.
6. 430·143168.
7. $777566·496.
8. $3598313.04.
9. 63073762.
10. 41281053.
11. 24294591.
12. $28047414.
13. 46350656.
14. 575630377.
15. 395494873.
16. 649435896.
17. 64008924.
18. 3704412744.
19. 403576660.
20. 175320.
21. $2941.64.
22. 2592.
23. 2303.
24. 3168.
25. $3243.75.
26. 4480.
27. $18.25.
28. 2144.
29. 81056.
30. 783.
31. 80 pages.
32. 1095.
33. 56940.
34. 768000.

EXERCISE 15.

1. 1632219$\frac{3}{8}$.
2. 12879585$\frac{4}{9}$.
3. 2087595$\frac{5}{12}$.
4. 3179483$\frac{4}{11}$.
5. 3569180$\frac{1}{8}$.
6. 1242017$\frac{14}{15}$.
7. 705323$\frac{1}{2}$.
8. 3458517$\frac{3}{10}$.
9. 1978148$\frac{1}{6}$.
10. 8370642$\frac{1}{24}$.

EXERCISE 16.

1. 30 quarts.
2. 86 cents.
3. 60 years.
4. 96 days.
5. \$3.40 and \$13.60.
6. 21 cents.

EXERCISE 17.

1. 6.
2. 5 for 40 cts., 7 for 56 cts.
3. 117 dimes.
4. 5 bbls.
5. 5 days.
6. 10 miles.

EXERCISE 18.

1. 6911$\frac{1}{4}$.
2. 137·52$\frac{4}{5}$.
3. 13281$\frac{1}{6}$.
4. \$115.17$\frac{1}{7}$.
5. \$95.53$\frac{3}{5}$.
6. 3186$\frac{2}{9}$.
7. \$64.26$\frac{8}{16}$.
8. 4206$\frac{1}{11}$.
9. 6368906.
10. \$53359.55$\frac{3}{8}$.
11. 1377181$\frac{1}{7}$.
12. \$71409.73$\frac{5}{9}$.
13. \$39064.06$\frac{4}{13}$.
14. 6859550.
15. 12667006$\frac{5}{8}$.
16. \$4780.66$\frac{7}{9}$.
17. 5894371$\frac{5}{9}$.
18. 28236344$\frac{1}{2}$.
19. 18824229$\frac{3}{8}$.
20. 14118·172$\frac{1}{4}$.
21. 11294537$\frac{3}{8}$.
22. 9412114$\frac{5}{8}$.
23. \$97.17$\frac{4}{7}$.
24. \$85.02$\frac{7}{8}$.
25. \$75.58$\frac{1}{9}$.
26. \$68.02$\frac{3}{16}$.
27. \$61.83$\frac{10}{11}$.

EXERCISE 19.

1. 6828$\frac{23}{54}$.
2. 4408$\frac{23}{33}$.
3. 10[illegible]$\frac{34}{72}$.
4. 1889$\frac{64}{108}$.
5. [illegible]309$\frac{88}{189}$.
6. 3450$\frac{76}{144}$.

EXERCISE 20.

1. 101$\frac{849}{857}$.
2. 167$\frac{296}{321}$.
3. 216$\frac{353}{403}$.
4. 127$\frac{636}{684}$.
5. 108·05$\underline{74}$.
6. 10·32$\underline{57}$.
7. ·95[illegible]$\underline{218}$.
8. 9902$\underline{383}$.
9. 7234$\underline{312}$.
10. ·7003$\underline{3749}$.
11. 8·57$\underline{1713}$.
12. ·03186$\underline{11}$.
13. 5098685000.
14. 25134919$\underline{1984}$
15. ·2587$\underline{1992}$.
16. 954118·46$\underline{2}$.
17. ·01061$\underline{9116}$.
18. 3753252$\underline{4}$.
19. 58264695$\underline{55}$.
20. 2[illegible]0302$\underline{14}$.
21. 1[illegible]·500001 &c.
[illegible] \$[illegible]0.82$\underline{7}$.
[illegible] 4[illegible]$\frac{2}{5}$.
24. 36.
25. 266$\frac{2}{3}$.
26. 2066666$\frac{38}{45}$.
27. 192268$\underline{340}$.
28. 925$\underline{35}$.

EXERCISE 21.

1. $317\frac{21}{32}$.
2. $160\frac{272}{373}$.
3. $261\frac{16}{202}$.
4. $127\frac{111}{711}$.
5. $551\frac{637}{1731}$.
6. $281\frac{616}{4281}$.
7. $133\frac{917}{3313}$.

EXERCISE 22.

1. 40 oranges.
2. 4 dimes.
3. 21 cents.
4. 24 oranges.
5. 24 oranges.
6. 4 men.

EXERCISE 23.

1. 63478.
2. 1420.
3. 5760.
4. 1094.
5. 24560.
6. 36136690704.
7. 6016.
8. 2472.
9. 20721.
10. $595834\frac{1}{4}$.
11. 52248.
12. 291454023.

EXERCISE 24.

1. 89 lbs. 3 oz.
2. £4[illegible] 1s.
3. 5 oz. 8 dwt. 15 gr.
4. 299 y[illegible] 2 nls.
5. 1199 bush.
6. 51 days, 20 [illegible] 57 min.
7. 7930 fathoms, [illegible] ft. 2 in.
8. 3934714880 ac.
   157388459520 r.
   6295538380800 per.
   190440036019200 yds.
   1713960324172800 ft.
   20568102866080883200 in.
9. 14 yds. 8 ft. 781 in.
10. 183332.
11. 1093 cds. 10 ft.
12. 298 ea. \$7. 1 dm. 4 c. 9 m.

EXERCISE 25.

1. \$1057.96$\frac{9}{12}$.
2. 350390.
3. \$990.54$\frac{9}{12}$.
4. \$1159.20.
5. \$298.00.
6. \$364.80.
7. \$178.20.
8. £3394 10s.
9. \$1778.65.
10. 1751 g. 18s.
11. 1[illegible]46 cr. 2s. 10d.
12. 113067 fourpences.
13. 9880 cr.
14. \$19538.
15. \$989.50.
16. 873740.
17. 57552.
18. 9621 $1\frac{1}{2}$d.
19. 63864160.
20. 33465 cr. 3s.
21. 118801 1s. 6d.
22. £69 $11\frac{3}{5}$d.
23. £182 10s. $8\frac{3}{5}$d.
24. 16048 2s.
25. 2890160.
26. 307608.
27. \$5126 .6$\frac{2}{3}$ cts.

EXERCISE 26.

1. £137 10s. 9½d.
2. 92 per. 4 yds. 2 ft.
3. 177 dys. 11 hrs. 28 min.
4. 17 cwt. 2 qrs. 7 lbs.
5. 15 lbs. 4 oz. 1 dwt. 14 gr.
6. 148 yds. 0 qrs. 1 nl.
7. 722 ac. 3 r. 21 per.
8. 120 yds. 23 ft. 119 in.
9. 9 cwt. 2 qrs.
10. 118 m. 5 f. 18 per. 3½ yds.
11. 175 qrs. 1 bus.
12. 29 a. 21 per.
13. 56 w. 2 d. 11 h. 16 min.
14. 15 lbs. 4 oz. 1 dwt. 14 gr.
15. 148 yds. 0 qrs. 2 nls.
16. £96 10s. 10d.
17. 16 tons, 107 lbs.
18. 6 hhd. 53 g. 3 qts.
19. 178 m. 35 per. 3 yd. 10 in.
20. 156 cds. 104 c. ft. 54 c. in.

EXERCISE 27.

1. £13 11s. 4d.
2. 78 bu. 1 gal.
3. 18 cwt. 3 lbs.
4. 83 yds. 3 qrs. 1 nl.
5. £13 17s. 8d.
6. 228 yds. gave $1482.

EXERCISE 28.

1. £24 11s. 7¾d.
2. 6 cwt. 2 qrs. 11 lbs.
3. 3 lbs. 8 oz. 19 dwt.
4. 4 ft. 30 p. 2½ yds.
5. 19 yds.
6. 25 a. 2 r. 25 p.
7. 192 t. 19 cwt. 3 qr. 6 lbs.
8. 3 lbs. 11 oz. 11 dwt. 6 gr.
9. 96 deg. 61½ m. 224 r. 3½ yd. 2 ft.
10. 19 yds.
11. 11 a. 3 r. 18 per.
12. 436 bu. 2 pk. 2 qt. 1 pt.
13. 2 w. 19 days.
14. $96.27½.
15. 48 cds. 106 ft. 58 in.
16. 3 yrs. 233 days.
17. 5 lbs. 4 dr. 1 scr. 16 gr.
18. 552 days.

EXERCISE 29.

1. 2s. 3d.
2. 11s. 9½d.
3. 15 yds. 2 qrs.
4. 45 gal. 1 qt.
5. 9 cwt. 1 qr. 15 lb.
6. £9 1s.
7. Subtract sub. from min.
8. Subtract rem. from min.
9. Add sub. to rem.
10. Subtract one from sum.
11. Subtract diff. from the greater.
12. Add diff. to less.
13. Half the sum + the diff. = greater, an half the sum—the diff = the less.

EXERCISE 30.

1. 19 cwt. 2 qrs. 8 lbs.
2. 194 lbs. 4 oz. 16 dwt.
3. 118 yds. 2 nls.
4. 25 ac. 2 r. 29 per.
5. 70 qrs. 2 bu. 2 pks.
6. 56 hrs. 33 min.
7. 670 a. 3 r. 24 per. $5\frac{1}{4}$ yd.
8. £1529 1s. $9\frac{3}{4}$d.
9. 1332 cds. 84 c. ft. 84 c. in.
10. 1951 ml. 3 fur. 37 per.
11. 124 gal. 1 pt.
12. 657 cwt. 2 qrs. 16 lbs.

EXERCISE 31.

2. 8432 lbs. 8 oz. 12 dwt.
3. £308478 16s. 4d.
4. 35737 m. 5 fur.
5. 295251 yd. 3 qr. $1\frac{1}{4}$ nl.
6. 24409 ac. 2 r. $17\frac{7}{8}$ per.
7. 22594 qr. $2\frac{5}{8}$ pks.
8. 7597 yrs. 31 w. $3\frac{1}{4}$ days.
9. 3345 cir. 4 signs.
10. 283 r. 6 qrs. $\frac{1}{3}$ sheets.
11. \$412164.81$\frac{5}{12}$.
12. \$552612.71$\frac{3}{7}$.
13. 33 oz.
14. 211 cwt. 3 qr. 1 lb. 4 oz.
15. 393 oz. 15 dwt.
16. 319 m. 1 fur. 30 per.
17. 9032 yds. 3 qrs. 2 nls.
18. 77612 m. 1 fur. 24 per.
19. 806 gal. 2 qt. 1 pt.
20. 47 hrs. 7 m. 30 sec.
21. 5991 gal.
22. \$1637.90.
23. 1534 g. 1 qt. 1 pt.
24. 84 cds. 60 ft. 144 in.
25. 748 ac. 38 per.

EXERCISE 32.

1. £6 12s.
2. £2 13s. 4d.
3. 14 cwt. 3 qrs. 13 lbs.
4. \$10·53.
5. 11 cents.

EXERCISE 33.

1. 6 cwt. 2 qrs. 12 lbs.
2. 1 lb. 1 oz. 2 dwt.
3. 1 per. 4 yds. $11\frac{1}{9}$ in.
4. 1 yd. 2 qrs. $0\frac{2}{11}$ nls.
5. 1 ac. $30\frac{7}{31}$ per.
6. 1 qr. $3\frac{4}{9}$ pks.
7. 11 cwt. 2 qrs. 11 lb. $11\frac{7}{11}$ oz.
8. £826 18s. $5\frac{39}{117}$d.
9. 5 gal. 3 qts. $\frac{3}{4}$ pt.
10. 5 per. 4 yd. 3 ft. 27 in.
11. 96 per. 22 yd. 7 ft. 24 in.
12. 5 F. ells. 2 qrs. 3 nls. $2\frac{1}{1088}$ in.
13. 24 d. 16 hs. 4 m. $8\frac{4}{9}$ sec.
14. 13 m. 5 fur. 18 per. $2\frac{41}{64}$ yds.

EXERCISE 35.

1. 1 cwt. 1 qr. $15\frac{5}{36}$ lbs.
2. 21 and 8 dwt.
3. 57200 times.
4. $472\ \frac{304}{1526}$ hhd.
5. 3 lb. 14 dwt. $13\frac{1}{3}$ gr.
6. 24 m. 7 fur. 4 per.
7. 47 yds.
8. 2s. $5\frac{421}{2071}$d.
9. $237\frac{2100}{2453}$ lots.
10. £1 2s. $4\frac{1}{2}$d.
11. 4 bu. 2 pks. 1 gal. $1\frac{13}{16}$ qt
12. 53 dys. 13 hrs. $42\frac{6}{7}$ min.
13. $94.50.
14. $302.40.
15. 1 hhd. 15 gal. 3 qts.
16. $1.80.
17. $2·215.
18. $3\frac{1}{4}$ qts.

EXERCISE 36.

1. 1003·87.
2. DXCLL, MMMMDCCIX, $\overline{\text{LXIX}}$, DXXXIV.
3. Fifty-nine whole numbers, and ten million nine hundred and sixty-seven thousand, nine hundred and ten one hundred millionths; one hundred and thirteen billion, three hundred and sixty-seven thousand one hundred and eighty-nine whole numbers, and one hundred and four thousand two hundred and fifty seven millionths; eighty-nine billion, seven hundred and fifty-four million six hundred and twenty-one thousand nine hundred and thirty-six; ten billion, one thousand whole numbers, and one thousand ten millionths.
4. £1423 11s. 3d.
5. $26.
6. 543478 lbs. 2 oz.
7. 1 oz. 12 dwt. $14\frac{515}{723}$ gr.
8. 55 ys. longest in England
9. £89 1s. 3d. in 1st.
   £68 17s. $11\frac{2}{3}$d. in 2nd.
   £20 16s. $7\frac{1}{3}$d. in 3rd & 4th.
10. $74.80.
11. 29 d. 12 h. 44 m. 3 sec.
12. 50 yds.
13. 629,097,707,498·000000617.
14. 11 per. 2 yds. 1.3995 ft.
15. (1.) 67950000·00.
    (2.) 6795·000000.
    (3.) 6·795000000.
    (4.) 6795·000000.
    (5.) 6795000·000.
    (6.) 679500000·0.
16. $58·03.
17. $67.50.
18. 25000 miles.
19. 5 ac. 3 r. $30\frac{4}{19}$ per.
20. $61\frac{1}{4}$ yds.
21. each woman $22.00.
    " man $66.00.

EXERCISE 36—continued.

22. each man 8 lb. $13\frac{31}{88}$ oz., cost £12 9s. $10\frac{13}{14}$d.
23. $1961\frac{1556}{1987}$ men.
$1377\frac{957}{1987}$ men.
24. 2 bu. 7 qts.
25. Factors 3, 7, 5, 6; remainders, 0, 0, 1, 4. $4 \times 5 + 1 \times 7 \times 3 = 441$.
26. 100 lbs.
27. $2\frac{1}{2}$ fur.
28. \$3·194.
29. (1.) 13 ac. 2 r. 17 per. 23 yds. $4\frac{3}{4}$ ft.
(2.) 9 ac. 2 r. 3 per. 29 yds. $2\frac{1}{4}$ ft. worth \$42570.06.
30. $6\frac{1}{2}$ qts.
31. 312 $\frac{78}{91}$ days.
32. 169 bu. 1 pk. 6 qts. 1 pt. $2\frac{6}{7}$ gills.
33. \$44 729.
34. 156 cds. 103 c. f. \$602.13.
35. Gain \$345.91.
36. £67 9s. $3\frac{9}{40}$d.
37. £63 6s. 8d.
38. $40\frac{25}{428}$ suits.
39. 29464 t. 5 cwt. 2 qr. 24 lb.
40. 13 persons.
41. 1032694 grains.
42. $1694\frac{4813}{9599}$.
43. 2827 w. 3 d. 16 h. 19 m.
44. 16640 times.
45. 36 $\frac{27}{35}$ oz.
46. 27.
47. ·246956 cords.
48. 11666·66, &c., sheets.
49. £19 5s.
50. £27 10s.
51. 14 cwt.
52. 72 cents.
53. \$1.92.
54. 1 pk. 4 qts.

EXERCISE 37.

1. 16.
2. 744 s. or \$148.80.
3. 3 cwt. 1 qr. 6 lbs.
4. 108 yds.
5. 120 dinners.
6. 4 coats.
7. 55 sheep.
8. 58 cwt. 1 qr. 20 lbs.
9. £4 1s., £10 9s. 3d.
10. $21\frac{2}{3}$ barrels.
11. 32 cents.
12. $425\frac{2}{3}$ barrels.
13. 23 yds. 1 qr. 2 nl.
14. $29535\frac{5}{17}$ and $141\frac{3}{17}$.
15. \$10.
16. 160 min. or 2 h. 40 min.
17. \$70. share of each, \$2.
18. $7\frac{3}{4}$ barrels.
19. $11\frac{1}{2}$ bush.
20. 16s.
21. \$35.
22. \$4·444.
23. $9\frac{3}{5}$ days.
24. $\frac{22}{45}$ of a dollar.

EXERCISE 38.

1. 112.
2. None.
3. 101.
4. 377.
5. 18.
6. 348.
7. 25.
8. 143.
9. 11.
10. 7.

EXERCISE 39.

1. 60.
2. 16.
3. 240.
4. 180.
5. 120.
6. 144.
7. 240.
8. 7560.
9. 1260.
10. 7200.

EXERCISE 41.

1. $32\frac{1}{16}$.
2. $8\frac{8}{9}$.
3. 9.
4. $1\frac{3}{14}$.
5. $9\frac{3648}{8288}$.
6. $3\frac{93}{183}$.

EXERCISE 42.

1. $\frac{116}{9}$.
2. $\frac{1643}{12}$.
3. $\frac{1131}{13}$.
4. $\frac{108}{16}$.
5. $\frac{1316}{17}$.
6. $\frac{3097}{17}$.

EXERCISE 43.

1. $\frac{72}{9}, \frac{88}{11}, \frac{136}{17}, \frac{96}{12}, \frac{288}{36}, \frac{320}{40}$.
2. $\frac{63}{9}, \frac{77}{11}, \frac{119}{17}, \frac{84}{12}, \frac{252}{36}, \frac{280}{40}$.
3. $\frac{108}{9}, \frac{132}{11}, \frac{204}{17}, \frac{144}{12}, \frac{432}{36}, \frac{480}{40}$.
4. $\frac{954}{9}, \frac{1166}{11}, \frac{1802}{17}, \frac{1272}{12}, \frac{3816}{36}, \frac{4240}{40}$.
5. $\frac{648}{9}, \frac{792}{11}, \frac{1224}{17}, \frac{864}{12}, \frac{2592}{36}, \frac{2880}{40}$.
6. $\frac{8172}{9}, \frac{9988}{11}, \frac{15436}{17}, \frac{10896}{12}, \frac{32688}{36}, \frac{36320}{40}$.

EXERCISE 44.

(1) $\frac{1597}{3701}$. (2) $\frac{9217}{16119}$. (3) $\frac{439}{1079}$.
(4) $\frac{1851}{2080}$. (5) $\frac{27}{2467}$. (6) $\frac{784}{1951}$.

1. $\frac{31}{32}$.
2. $\frac{865}{4}$.
3. $\frac{99}{8}$.
4. $\frac{1053}{80}$.
5. $\frac{21}{4}$.
6. $\frac{9}{1300}$.